

White

**Rose
Maths**

Spring - Block 3

Algebra

Overview

Small Steps

- Find a rule – one step
- Find a rule – two step
- Forming expressions
- Substitution
- Formulae
- Forming equations
- Solve simple one-step equations
- Solve two-step equations
- Find pairs of values
- Enumerate possibilities

NC Objectives

Use simple formulae.

Generate and describe linear number sequences.

Express missing number problems algebraically.

Find pairs of numbers that satisfy an equation with two unknowns.

Enumerate possibilities of combinations of two variables.

Find a Rule – One Step

Notes and Guidance

Children explore simple one-step function machines. Explain that a one-step function is where they perform just one operation on the input.

Children understand that for each number they put into a function machine, there is an output. They should also be taught to “work backwards” to find the input given the output. Given a set of inputs and outputs, they should be able to work out the function.

Mathematical Talk

What do you think “one-step function” means?

What examples of functions do you know?

Do some functions have more than one name?

What do you think input and output mean?

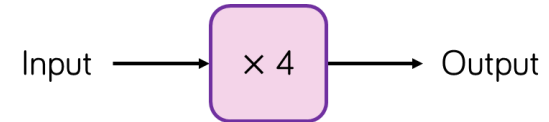
What is the output if?

What is the input if?

How many sets of inputs and outputs do you need to be able to work out the function? Explain how you know.

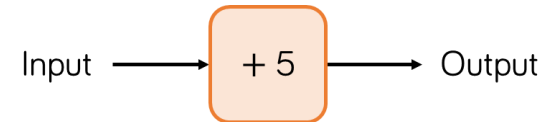
Varied Fluency

Here is a function machine.



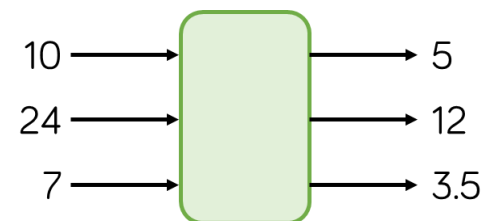
- What is the output if the input is 2?
- What is the output if the input is 7.2?
- What is the input if the output was 20?
- What is the input if the output was 22?

Complete the table for the function machine.



Input	5	5.8	10	- 3	- 8			
Output						9	169	0

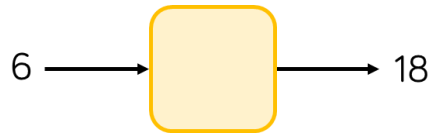
Find the missing function.



Find a Rule – One Step

Reasoning and Problem Solving

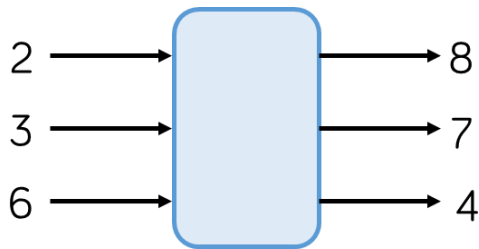
Eva has a one-step function machine. She puts in the number 6 and the number 18 comes out.



What could the function be?
How many different answers can you find?

The function could be $+ 12$, $\times 3$

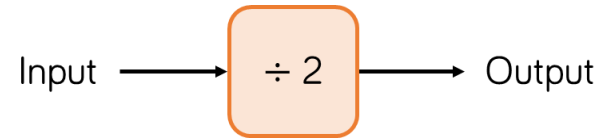
Amir puts some numbers into a function machine.



What is the output from the function when the input is 16?

The function is subtract from 10 so the output is -6

Dora puts a number into the function machine.



Dora's number is:

- A factor of 32
- A multiple of 8
- A square number

What is Dora's input?
What is her output?

Can you create your own clues for the numbers you put into a function machine for a partner to solve?

Dora's input is 16
Her output is 8

Find a Rule – Two Step

Notes and Guidance

Children build on their knowledge of one-step functions to look at two-step function machines. Discuss with children whether a function such as $+ 5$ and $+ 6$ is a two-step function machine or whether it can be written as a one-step function.

Children look at strategies to find the functions. They can use trial and improvement or consider the pattern of differences.

Children record their input and output values in the form of a table.

Mathematical Talk

How can you write $+ 5$ followed by $- 2$ as a one-step function?

If I change the order of the functions, is the output the same?

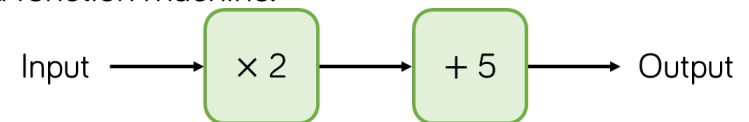
What is the output if?

What is the input if?

If you add 3 to a number and then add 5 to the result, how much have you added on altogether?

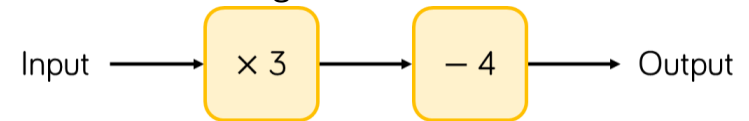
Varied Fluency

Here is a function machine.



- What is the output if the input is 5?
- What is the input if the output is 19?
- What is the output if the input is 3.5?

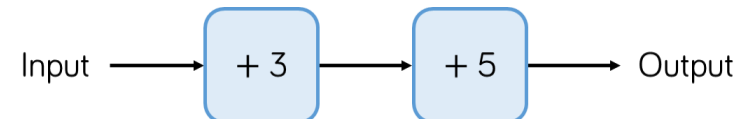
Complete the table for the given function machine.



Input	1	2	3	4	5
Output					

- What patterns do you notice in the outputs?
- What is the input if 20 is the output? How did you work it out?

How can you write this two-step machine as a one-step machine?

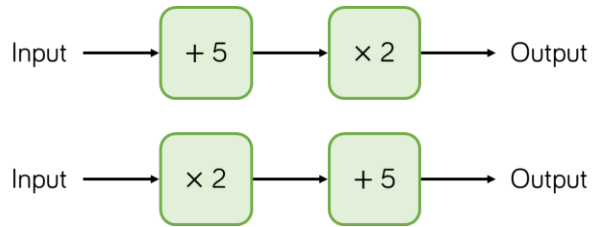


Check your answer by inputting values.

Find a Rule – Two Step

Reasoning and Problem Solving

Teddy has two function machines.



He says,



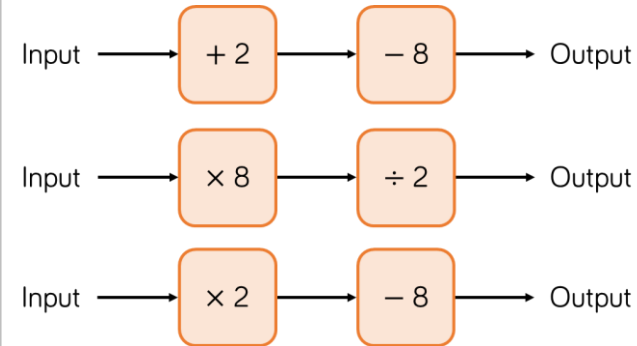
The function machines will give the same answer.

Is Teddy correct?

Is there an input that will give the same output for both machines?

No they do not give the same answer. Encourage children to refer to the order of operations to help them understand why the outputs are different.

Mo has the following function machines.



The first one can be written as $- 6$

The second can be written as $\times 4$

The third cannot be written as a single machine.

Explain which of these can be written as single function machines.

Forming Expressions

Notes and Guidance

Children have now met one-step and two-step function machines with numerical inputs.

In this step, children use simple algebraic inputs e.g. y . Using these inputs in a function machine leads them to forming expressions e.g. $y + 4$. The use of cubes to represent a variable can aid understanding.

Children are introduced to conventions that we use when writing algebraic expressions. e.g. $y \times 4$ as $4y$.

Mathematical Talk

What expressions can be formed from this function machine?

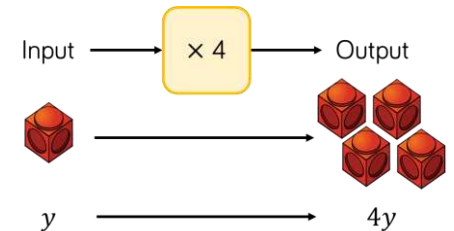
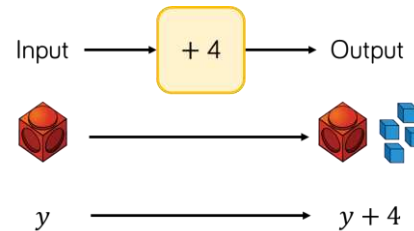
What would the function machine look like for this rule/expression?

How can you write $x \times 3 + 6$ differently?

Are $2a + 6$ and $6 + 2a$ the same? Explain your answer

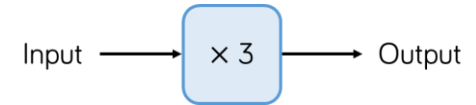
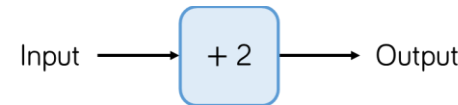
Varied Fluency

Mo uses cubes to write expressions for function machines.

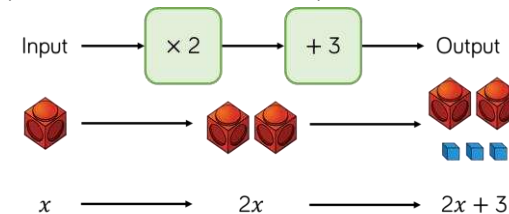


Use Mo's method to represent the function machines.

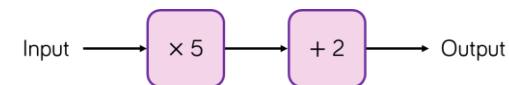
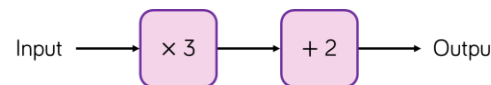
What is the output for each machine when the input is a ?



Eva is writing expressions for two-step function machines.



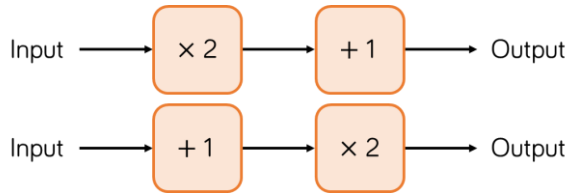
Use Eva's method to write expressions for the function machines.



Forming Expressions

Reasoning and Problem Solving

Amir inputs m into these function machines.



He says the outputs of the machines will be the same.

Do you agree?

Explain your answer.

No, because $2m + 1$ isn't the same as $2m + 2$

$2m + 1$

Input



$\times 2$



$+ 1$



$2m + 2$

Input



$+ 1$

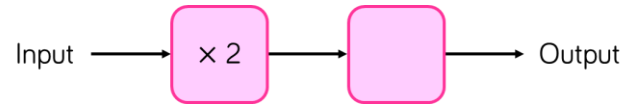


$\times 2$



Children may use examples with numbers to show this.

This function machine gives the same output for every input. For example if the input is 5 then the output is 5 and so on.



What is the missing part of the function?

What other pairs of functions can you think that will do the same?

$\div 2$

Other pairs of functions that will do the same are functions that are the inverse of each other e.g. $+ 3, - 3$

Substitution

Notes and Guidance

Children substitute into simple expressions to find a particular value.

They have already experienced inputting into a function machine, and teachers can make the links between these two concepts.

Children will need to understand that the same expression can have different values depending on what has been substituted.

Mathematical Talk

Which letter represents the star?

Which letter represents the heart?

Would it still be correct if it was written as $a + b + c$?

What does it mean when a number is next to a letter?

Is $a + b + b$ the same as $a + 2b$?

Varied Fluency

■ If $\star = 7$ and $\heartsuit = 5$, what is the value of:

$$\star + \heartsuit + \heartsuit$$

If $a = 7$ and $b = 5$ what is the value of:

$$a + b + b$$

What is the same and what is different about this question?

■ Substitute the following to work out the values of the expressions.

$$w = 3 \quad x = 5 \quad y = 2.5$$

- $w + 10$
- $w + x$
- $y - w$

■ Substitute the following to work out the values of the expressions.

$$w = 10 \quad x = \frac{1}{4} \quad y = 2.5$$

- $3y$
- wx
- $12 + 8.8w$
- $wy + 4x$

Substitution

Reasoning and Problem Solving

Here are two formulae.

$$p = 2a + 5$$

$$c = 10 - p$$

Find the value of c when $a = 10$

$$c = -15$$

$$x = 2c + 6$$

Whitney says,



$x = 12$ because c must be equal to 3 because it's the 3rd letter in the alphabet

Is Whitney correct?

Amir says,

When $c = 5$, $x = 31$



Amir is wrong.

Explain why.

What would the correct value of x be?

No Whitney is incorrect. c could have any value.

Amir has put the 2 next to the 5 to make 25 instead of multiplying 2 by 5

The correct value of x would be 16

Formulae

Notes and Guidance

Children substitute into familiar formulae such as those for area and volume.

They also use simple formulae to work out values of everyday activities such as the cost of a taxi or the amount of medicine to take given a person's age.

Mathematical Talk

What tells you something is a formula?

Which of the rectangles is the odd one out? Why?

Could you write the formula for a rectangle in a different way?

What other formulae do you know?

Varied Fluency

Which of the following is a formula?

$$P = 2l + 2w$$

$$3d + 5$$

$$20 = 3x - 2$$

Explain why the other two are not formulae.

Eva uses the formula $P = 2l + 2w$ to find the perimeter of rectangles.

Use this formula to find the perimeter of rectangles with the following lengths and widths.

- $l = 15, w = 4$
- $l = \frac{1}{4}, w = \frac{3}{8}$
- $l = w = 5.1$

This is the formula to work out the cost of a taxi.

$$C = 1.50 + 0.3m$$

C = the cost of the journey in £

m = number of miles travelled.

Work out the cost of a 12-mile taxi journey

Formulae

Reasoning and Problem Solving

Jack and Dora are using the following formula to work out what they should charge for four hours of cleaning.

$$\text{Cost in pounds} = 20 + 10 \times \text{number of hours}$$

Jack thinks they should charge £60

Dora thinks they should charge £120

Who do you agree with?

Why?

Jack is correct as multiplication should be performed first following the order of operations.

Dora has not used the order of operations - she has added 20 and 10 and then multiplied 30 by 4

The rule for making scones is use 4 times as much flour (f) as butter (b).

Which is the correct formula to represent this?

A

$$f = \frac{b}{4}$$

B

$$f = 4b$$

C

$$f = b + 4$$

D

$$4f = b$$

Explain why the others are incorrect.

B is correct.

A shows the amount of flour is a quarter of the amount of butter.

C shows the amount of flour is 4 more than butter.

D shows butter is 4 times the amount of flour.

Forming Equations

Notes and Guidance

Building on the earlier step of forming expressions, children now use algebraic notation to form one-step equations. They need to know the difference between an expression like $x + 5$, which can take different values depending on the value of x , and an equation like $x + 5 = 11.2$ where x is a specific unknown value. This is best introduced using concrete materials e.g. cubes, can be used to represent the unknown values with counters being used to represent known numbers.

Mathematical Talk

What does the cube represent?
What do the counters represent?




Design your own 'think of a number' problems.

What's the difference between an expression and an equation?

What's the difference between a formula and an equation?

Varied Fluency

Amir represents a word problem using cubes, counters and algebra.

Words	Concrete	Algebra
I think of a number		x
Add 3		$x + 3$
My answer is 5		$x + 3 = 5$

Complete this table using Amir's method.

Words	Concrete	Algebra
I think of a number		
Add 1		
My answer is 8		

- A book costs £5 and a magazine costs £ n
The total cost of the book and magazine is £8
Write this information as an equation.
- Write down algebraic equations for these word problems.
 - I think of a number, subtract 17, my answer is 20
 - I think of a number, multiply it by 5, my answer is 45

Forming Equations

Reasoning and Problem Solving

Rosie thinks of a number. She adds 7 and divides her answer by 2

Teddy thinks of a number. He multiplies by 3 and subtracts 4

Rosie and Teddy think of the same number.

Rosie's answer is 9

What is Teddy's answer?

Rosie and Teddy think of the same number again. This time, they both get the same answer.

Use trial and improvement to find the number they were thinking of.

They both think of 11, therefore Teddy's answer is 29

They think of 3 and the answer they both get is 5

Eva spends 92p on yo-yos and sweets

She buys y yo-yos costing 11p and s sweets costing 4p.

Can you write an equation to represent what Eva has bought?

How many yo-yos and sweets could Eva have bought?

Can you write a similar word problem to describe this equation?

$$74 = 15t + 2m$$

$$92 = 11y + 4s$$

She could have bought 1 sweet and 8 yo-yos or 4 yo-yos and 12 sweets.

One-step Equations

Notes and Guidance

Children solve simple one step equations involving the four operations.

Children should explore this through the use of concrete materials such as cubes, counters and cups.

It is recommended that children learn to solve equations using a balancing method using inverse operations.

Mathematical Talk

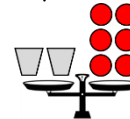
Can you make some of your own equations using cups and counters for a friend to solve?

Why do you think the equation is set up on a balance? What does the balance represent? How does this help you solve the equation?

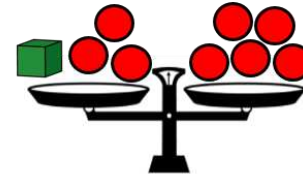
What is the same and what is different about each bar model?

Varied Fluency

- How many counters is each cup worth?
Write down and solve the equation represented by the diagram.



- Solve the equation represented on the scales.
Can you draw a diagram to go with the next step?

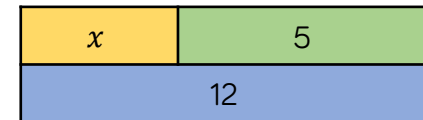
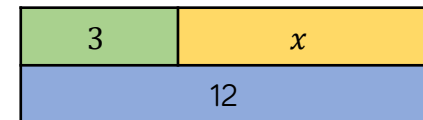


- Match each equation to the correct bar model and then solve to find the value of x .

$$x + 5 = 12$$

$$3x = 12$$

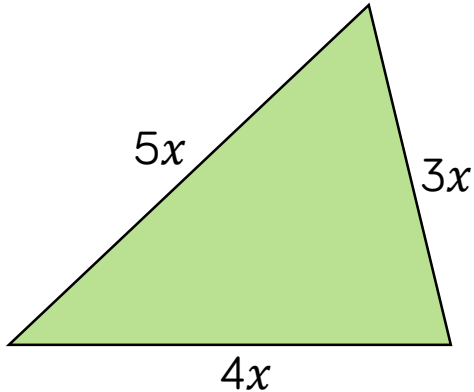
$$12 = 3 + x$$



One-step Equations

Reasoning and Problem Solving

The perimeter of the triangle is 216 cm.



Form an equation to show this information.

Solve the equation to find the value of x .

Work out the lengths of the sides of the triangle.

$$\begin{aligned} 3x + 4x + 5x &= 216 \\ 12x &= 216 \\ x &= 18 \end{aligned}$$

$$\begin{aligned} 5 \times 18 &= 90 \\ 3 \times 18 &= 54 \\ 4 \times 18 &= 72 \end{aligned}$$

- Hannah is 8 years old
- Jack is 13 years old
- Grandma is $x + 12$ years old.
- The sum of their ages is 100

Form and solve an equation to work out how old Grandma is.

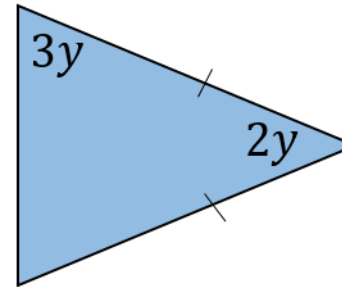
$$\begin{aligned} 8 + 13 + x + 12 &= 100 \end{aligned}$$

$$33 + x = 100$$

$$x = 77$$

Grandma is 77 years old.

What is the size of the smallest angle in this isosceles triangle?



How can you check your answer?

$$\begin{aligned} 8y &= 180 \\ y &= 22.5 \end{aligned}$$

Smallest angle = 45°

Check by working them all out and see if they add to 180°

Two-step Equations

Notes and Guidance

Children progress from solving equations that require one-step to equations that require two steps.

Children should think of each equation as a balance and solve it through doing the same thing to each side of the equation.

This should be introduced using concrete and pictorial methods alongside the abstract notation as shown. Only when secure in their understanding should children try this without the support of bar models or similar representations.

Mathematical Talk

Why do you have to do the same to each side of the equation?







Why subtract 1? What does this do to the left hand side of the equation?

Does the order the equation is written in matter?

What's the same and what's different about solving the equations $2x + 1 = 17$ and $2x - 1 = 17$?

Varied Fluency

Here is each step of an equation represented with concrete resources.

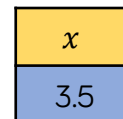
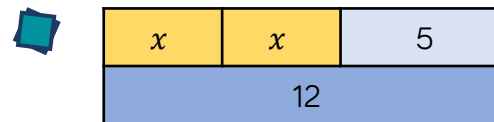
	=		$2x + 1 = 5$
		-1	-1
	=		$2x = 4$
		$\div 2$	$\div 2$
	=		$x = 2$

Use this method to solve:

$$4y + 2 = 6$$

$$9 = 2x + 5$$

$$1 + 5a = 16$$



Here is each step of an equation represented by a bar model. Write the algebraic steps that show the solution of the equation.

Use bar models to solve these equations.

$$3b + 4 = 19$$

$$20 = 4b + 2$$

Two-step Equations

Reasoning and Problem Solving

The length of a rectangle is $2x + 3$
 The width of the same rectangle is $x - 2$
 The perimeter is 17 cm.

Find the area of the rectangle.

$$\begin{aligned} 6x + 2 &= 17 \\ 6x &= 15 \\ x &= 2.5 \\ \text{Length} &= 8 \text{ cm} \\ \text{Width} &= 0.5 \text{ cm} \\ \text{Area} &= 4 \text{ cm}^2 \end{aligned}$$

Alex has some algebra expression cards.



$$y + 4$$

$$2y$$

$$3y - 1$$

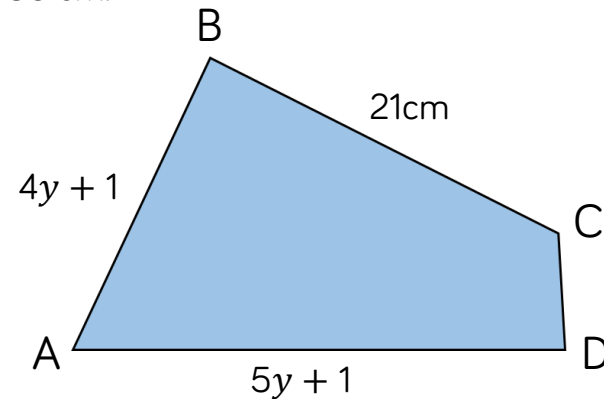
The mean of the cards is 19
 Work out the value of each card.

$$\begin{aligned} 6y + 3 &= 57 \\ 6y &= 54 \\ y &= 9 \end{aligned}$$

Card values:
 13
 18
 26

Here is the quadrilateral ABCD.

The perimeter of the quadrilateral is 80 cm.



AB is the same length as BC.

Find the length of CD.

$$\begin{aligned} 4y + 1 &= 21 \\ 4y &= 20 \\ y &= 5 \end{aligned}$$

$$\begin{aligned} AB &= 21 \text{ cm} \\ BC &= 21 \text{ cm} \\ AD &= 26 \text{ cm} \\ CD &= 80 - (21 + 21 + 26) = 12 \text{ cm} \end{aligned}$$

Find Pairs of Values (1)

Notes and Guidance

Children use their understanding of substitution to consider what possible values a pair of variables can take.

At this stage we should focus on integer values, but other solutions could be a point for discussion.

Children can find values by trial and improvement, but should be encouraged to work systematically.

Mathematical Talk


Can a and b be the same value?

Is it possible for a or b to be zero?

How many possible integer answers are there? Convince me you have them all.

What do you notice about the values of c and d ?

Varied Fluency

 a and b are variables:

$$a + b = 6$$

There are lots of possible solutions to This equation.

Find 5 different possible integer values for a and b .

a	b

 X and Y are whole numbers.

- X is a one digit odd number.
- Y is a two digit even number.
- $X + Y = 25$

Find all the possible pairs of numbers that satisfy the equation.



$$c \times d = 48$$

What are the possible integer values of c and d ?
How many different pairs of values can you find?

Find Pairs of Values (1)

Reasoning and Problem Solving

a , b and c are integers between 0 and 5

$$\begin{aligned}a + b &= 6 \\ b + c &= 4\end{aligned}$$

Find the values of a , b and c

How many different possibilities can you find?

Possible answers:

$$\begin{aligned}a &= 4 & b &= 2 \\ c &= 2\end{aligned}$$

$$\begin{aligned}a &= 3 & b &= 3 \\ c &= 1\end{aligned}$$

$$\begin{aligned}a &= 2 & b &= 4 \\ c &= 0\end{aligned}$$

x and y are both positive whole numbers.

$$\frac{x}{y} = 4$$

Dora says,



x will always be a multiple of 4

Jack says,



y will always be a factor of 4

Only one is correct – who is it?
Explain your answer.

Possible answer:

Dora is correct as x will always have to divide into 4 equal parts e.g.
 $32 \div 8 = 4$,
 $16 \div 4 = 4$

Jack is incorrect.
 $40 \div 10 = 4$ and
10 is not a factor of 4

Find Pairs of Values (2)

Notes and Guidance

Building on from the last step, children find possible solutions to equations which involve multiples of one or more unknown.

They should be encouraged to try one number for one of the variables first and then work out the corresponding value of the other variable. Children should then work systematically to test if there are other possible solutions that meet the given conditions.

Mathematical Talk

What does $2a$ mean? (2 multiplied by an unknown number)
What is the greatest/smallest number 'a' can be?

What strategy did you use to find the value of 'b'?

Can you draw a bar model to represent the following equations:

$$3f + g = 20$$

$$7a + 3b = 40$$

What could the letters represent?

Varied Fluency

- In this equation, a and b are both whole numbers which are less than 12.

$$2a = b$$

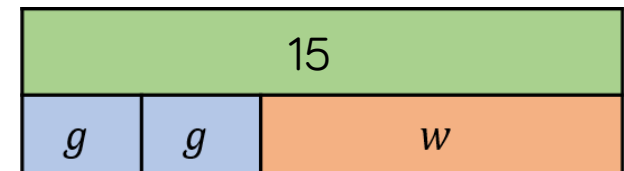
Write the calculations that would show all the possible values for a and b .

- Chose values of x and use the equation to work out the values of y .

$$7x + 4 = y$$

Value of x	Value of y

- $2g + w = 15$
 g and w are positive whole numbers.
Write down all the possible values for g and w , show each of them in a bar model.



Find Pairs of Values (2)

Reasoning and Problem Solving

$$ab + b = 18$$

Mo says,



a and b must both be odd numbers

Is Mo correct?
Explain your answer.

Possible answer:

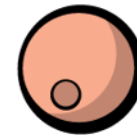
Mo is incorrect. Children may give examples to prove Mo is correct e.g. if $a = 5$ and $b = 3$, but there are also examples to show he is incorrect e.g. $a = 2$ and $b = 6$ where a and b are both even.

Large beads cost 5p and small beads cost 4p

Rosie has 79p to spend on beads.



4p



5p

How many different combinations of small and large beads can Rosie buy?

Can you write expressions that show all the solutions?

Possible answers:

$$3l + 16s$$

$$7l + 11s$$

$$11l + 6s$$

$$15l + s$$