## White <br> Autumn - Block 4 <br> Multiplication \& Division

## Overview

## Small Steps

## NC Objectives

Multiply and divide numbers mentally drawing upon known facts.

Multiply and divide whole numbers by 10,100 and 1,000

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Factors
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## Common factors

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Prime numbers
Square numbers
Cube numbers
Multiply by 10, 100 and 1,000
Divide by 10, 100 and 1,000
Multiples of 10, 100 and 1,000
Multiples
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Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.

Recognise and use square numbers and cube numbers and the notation for squared ( ${ }^{2}$ ) and cubed ( ${ }^{3}$ )

Solve problems involving multiplication and division including using knowledge of factors and multiples, squares and cubes.

Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

Establish whether a number up to 100 is prime and recall prime numbers up to 19

## Multiples

## Notes and Guidance

Building on their times tables knowledge, children will find multiples of whole numbers. Children build multiples of a number using concrete and pictorial representations e.g. an array. Children understand that a multiple of a number is the product of the number and another whole number.

Multiplying decimal numbers by 10, 100 and 1,000 forms part of Year 5 Summer block 1.

## Mathematical Talk

What do you notice about the multiples of 5 ? What is the same about each of them, what is different?

Look at multiples of other numbers, is there a pattern that links them to each other?

Are all multiples of 8 multiples of 4 ?
Are all multiples of 4 multiples of 8 ?

## Varied Fluency

Circle the multiples of 5
$25 \quad 32 \quad 54 \quad 175 \quad 554 \quad 3000$

What do you notice about the multiples of 5 ?
7,135 is a multiple of 5. Explain how you know.
Roll 2 dice (1-6), and multiply the numbers the you roll. List all the numbers that this number is a multiple of.
Repeat the dice roll.
Use a table to show your results.
Multiply the numbers you roll to complete the table.

## Multiples

## Reasoning and Problem Solving

Use 0-9 digit cards. Choose 2 cards and multiply the digits shown.

What is your number a multiple of?
Is it a multiple of more than one number?

Find all the numbers you can make using the digit cards.

Use the table below to help.

|  | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  |  |  |  |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |  |  |  |  |
| 6 |  |  |  |  |  |  |  |  |  |  |
| 7 |  |  |  |  |  |  |  |  |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |
| 9 |  |  |  |  |  |  |  |  |  |  |

## Always, Sometimes, Never

- The product of two even numbers is a multiple of an odd number.
- The product of two odd numbers is a multiple of an even number.

Eva's age is a multiple of 7 and is 3 less than a multiple of 8

She is younger than 40
How old is Eva?

Always - all integers are multiples of 1 , which is an odd number.

Never - Two odd numbers multiplied together are always a multiple of an odd number.

Eva is 21 years old.

## Factors

## Notes and Guidance

Children understand the relationship between multiplication and division and use arrays to show the relationship between them. Children learn that factors of a number multiply together to give that number, meaning that factors come in pairs. Factors are the whole numbers that you multiply together to get another whole number (factor $\times$ factor $=$ product).

## Mathematical Talk

How can you work in a systematic way to prove you have found all the factors?

Do factors always come in pairs?
How can we use our multiplication and division facts to find factors?

## Varied Fluency

If you have twenty counters, how many different ways of arranging them can you find?


How many factors of twenty have you found by arranging your counters in different arrays?
$\square$ Circle the factors of 60

$$
9,6,8,4,12,5,60,15,45
$$

Which factors of 60 are not shown?

Fill in the missing factors of 24
$\qquad$
$3 \times$ $\qquad$
$\qquad$ $\times$ $\qquad$
What do you notice about the order of the factors?
Use this method to find the factors of 42

## Factors

## Reasoning and Problem Solving

Here is Annie's method for finding factor pairs of 36

| 1 | 36 |
| :---: | :---: |
| 2 | 18 |
| 3 | 12 |
| 4 | 9 |
| 5 | $X$ |
| 6 | 6 |

When do you put a cross next to a number?

How many factors does 36 have?

Use Annie's method to find all the factors of 64

If it is not a factor, put a cross.

36 has 9 factors.

Factors of 64:

| 1 | 64 |
| :---: | :---: |
| 2 | 32 |
| 3 | $x$ |
| 4 | 16 |
| 5 | $x$ |
| 6 | $x$ |
| 7 | $x$ |
| 8 | 8 |

## Always, Sometimes, Never

- An even number has an even amount of factors.
- An odd number has an odd amount of factors.


## True or False?

The bigger the number, the more factors it has.

Sometimes, e.g. 6 has four factors
but 36 has nine.
Sometimes, e.g. 21 has four factors but 25 has three.

False. For example, 12 has 6 factors but 13 only has 2

## Common Factors

## Notes and Guidance

Using their knowledge of factors, children find the common factors of two numbers.

They use arrays to compare the factors of a number and use Venn diagrams to show their results.

## Mathematical Talk

How can we find the common factors systematically?

Which number is a common factor of a pair of numbers?

How does a Venn diagram help to show common factors? Where are the common factors?

## Varied Fluency

Use arrays to find the common factors of 12 and 15 Can we arrange each number in counters in one row?

Yes- so they have a common factor of one.
Can we arrange each number in counters in two equal rows?

## 00000000000000

We can for 12 , so 2 is a factor of 12 , but we can't for 15 , so 2 is not a factor of 15 , meaning 2 is not a common factor of 12 and 15
Continue to work through the factors systematically until you find all the common factors.
$\square$ Fill in the Venn diagram to show the factors of 20 and 24


Where are the common factors of 20 and 24 ?
Use a Venn diagram to show the common factors of 9 and 15

## Common Factors

## Reasoning and Problem Solving

## True or False?

- 1 is a factor of every number.
- 1 is a multiple of every number.
- 0 is a factor of every number.
- 0 is a multiple of every number.

| True |
| :--- |
| False |
| True |

I am thinking of two 2-digit numbers.
24 and 60
Both of the numbers have a digit total of six.

Their common factors are:

$$
1,2,3,4,6 \text {, and } 12
$$

What are the numbers?

## Prime Numbers

## Notes and Guidance

Using their knowledge of factors, children see that some numbers only have two factors. They are taught that these are numbers called prime numbers, and that non-primes are called composite numbers. Children can recall primes up to 19 and are able to establish whether a number is prime up to 100. Using primes, they break a number down into its prime factors. Children learn that 1 is not a prime number because it does not have exactly two factors (it only has 1 factor).

## Mathematical Talk

How many factors does each number have?
How many other numbers can you find that have this number of factors?

What is a prime number?
What is a composite number?
How many factors does a prime number have?

## Varied Fluency

Use counters to find the factors of the following numbers.

$$
5, \quad 13, \quad 17,23
$$

What do you notice about the arrays?
A prime number has exactly 2 factors, one and itself. A composite number can be divided by numbers other than 1 and itself to give a whole number answer.
Sort the numbers into the table.


Put two of your own numbers into the table.
Why are two of the boxes empty?
Would 1 be able to go in the tablet? Why or why not?

## Prime Numbers

## Reasoning and Problem Solving

Find all the prime numbers between 10 and 100 , sort them in the table below.

| End in a 1 | End in a 3 | End in a 7 | End in a 9 |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |

Why do no two-digit prime numbers end in an even digit?

Why do no two-digit prime numbers end in a 5 ?

| End in a 1 | End in a 3 |
| :---: | :---: |
| $11,31, ~ 41, ~$ <br> 61,71, | $13,23,43$, <br> 53,73 |
| End in a 7 | End in a 9 |
| 17,37, | $19,29,59$, |
| 47,67, | 79,89 |
| 97 |  |

Because all twodigit even numbers have more than 2 factors.

Because all twodigit numbers ending in 5 are divisible by 5 as well as 1 and itself, so have more than 2 factors.

Dora says all prime numbers have to be odd.


Her friend Amir says that means all odd numbers are prime, so 9,27 and 45 are prime numbers.


Explain Amir's and Dora's mistakes and correct them.

Dora is incorrect because 2 is a prime number (it has exactly 2 factors).

Amir thinks all odd numbers are
prime but he is incorrect because most odd numbers have more than 2 factors.

## E.g.

Factors of 9:
1,3 and 9
Factors of 27:
$1,3,9$ and 27

## Square Numbers

## Notes and Guidance

Children will need to be able to find factors of numbers. Square numbers have an odd number of factors and are the result of multiplying a whole number by itself.

Children learn the notation for squared is
2

## Mathematical Talk

Why are square numbers called 'square' numbers?

Are there any patterns in the sequence of square numbers?

Are the squares of even numbers always even?

## Varied Fluency

What does this array show you?
Why is this array square?


How many ways are there of arranging 36 counters in an array?
What is the same about each array?
What is different?

Find the first 12 square numbers.
Show why they are square numbers.
How many different squares can you make using counters?
What do you notice?
Are there any patterns?

Are the squares of odd numbers always odd?

## Square Numbers

## Reasoning and Problem Solving

| Teddy says, <br> Factors come in pairs so all numbers must have an even number of factors. <br> Do you agree? <br> Explain your reasoning. | No. <br> Square numbers have an odd number of factors (e.g. the factors of 25 are 1, 25 and 5). |
| :---: | :---: |
| How many square numbers can you make by adding prime numbers together? <br> Here's one to get you started: $2+2=4$ | Solutions include: $\begin{aligned} & 2+2=4 \\ & 2+7=9 \\ & 11+5=16 \\ & 23+2=25 \\ & 29+7=36 \end{aligned}$ |


| Whitney thinks that $4^{2}$ is equal to 16 | Children may use <br> concrete materials |
| :--- | :--- |
| Do you agree? |  |
| Convince me. | or draw pictures to <br> prove it. <br> Children should <br> spot that 6 has <br> been multiplied by 2 |
| Amir thinks that $6^{2}$ is equal to 12 |  |
| Do you magree? create |  |
| the array to prove |  |
| that $6^{2}=36$ and 6 |  |
| $\times 2=12$ |  |$|$| Never. Square |
| :--- |
| numbers have an |
| odd number of |
| factors because one |
| of their factors does |
| not have a pair. |

## Cube Numbers

## Notes and Guidance

Children learn that a cube number is the result of multiplying a whole number by itself three times e.g. $6 \times 6 \times 6$

If you multiply a number by itself, then itself again, the result is a cube number.

Children learn the notation for cubed is

## Mathematical Talk

Why are cube numbers called 'cube' numbers?
How are squared and cubed numbers similar?
How are they different?
True or False: cubes of even numbers are even and cubes of odd numbers are odd.

## Varied Fluency

Use multilink cubes to investigate how many are needed to make different sized cubes.

How many multilink blocks are required to make the first cube number? The second? Third?

Can you predict what the tenth cube number is going to be?
$\square$ Complete the table.

|  |  | 8 |
| :---: | :---: | :---: |
| $3^{3}$ | $3 \times 3 \times 3$ | 27 |
| $4^{3}$ |  |  |
| $5^{3}$ | $5 \times 5 \times 5$ |  |
|  | $6 \times 6 \times 6$ |  |

Calculate:

$$
\begin{array}{rr}
4^{3}= & 5^{3}= \\
3 \text { cubed }= & 6 \text { cubed }=
\end{array}
$$

## Cube Numbers

## Reasoning and Problem Solving

| Rosie says, <br> Do you agree? <br> Explain your answer. | Rosie is wrong, she has multiplied 5 by 3 rather than by itself 3 times. $\begin{aligned} & 5^{3}=5 \times 5 \times 5 \\ & 5 \times 5 \times 5=125 \end{aligned}$ |
| :---: | :---: |
| Here are 3 cards $\square$ $\square$ $\square$ <br> On each card there is a cube number. Use these calculations to find each number. $\begin{gathered} A \times A=B \\ B+B-3=C \\ \text { Digit total of } C=A \end{gathered}$ | $\begin{aligned} & A=8 \\ & B=64 \\ & C=125 \end{aligned}$ |


| Dora is thinking of a two-digit number <br> that is both a square and a cube number. <br> What number is she thinking of? | 64 |
| :--- | :--- |
| Teddy's age is a cube number. | 8 years old |
| Next year his age will be a square <br> number. |  |
| How old is he now? | 125 and 25 |
| The sum of a cube number and a square <br> number is 150 <br> What are the two numbers? |  |

## Multiply by 10, 100 and 1,000

## Notes and Guidance

## Varied Fluency

Make 234 on a place value grid using counters.

| HTh | TTh | Th | H | T | O |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\bigcirc$ | $\bigcirc \bigcirc$ | $\bigcirc \bigcirc$ |
|  |  |  | $\bigcirc$ | $\bigcirc$ | $\bigcirc \bigcirc$ |

When I multiply 234 by 10 , where will I move my counters? Is this always the case when multiplying by $10 ?$

Complete the following questions using counters and a place value grid.

$$
\begin{array}{cl}
234 \times 100= & =324 \times 100 \\
100 \times 36=- & -=300 \times 207= \\
45,020 \times 10= & -, 406 \times 1,000
\end{array}
$$

Use <, > or = to complete the statements.
$71 \times 1,000$
$100 \times 32$
$48 \times 100$
$16 \times 1,000$
$48 \times 10 \times 10 \times 10$

## Multiply by 10, 100 and 1,000

## Reasoning and Problem Solving

| Rosie has $£ 300$ in her bank account. <br> Tommy has 100 times more than Rosie in his bank account. <br> How much more money does Tommy have than Rosie? | Tommy has £30,000 <br> Tommy has £29,700 more than Rosie. |
| :---: | :---: |
| Whitney has $£ 1,020$ in her bank account. <br> Tommy has £120 in his bank account. <br> Is Whitney correct? Explain your reasoning. | Whitney is incorrect, she would need to have $£ 1,200$ if this were the case (Or Tommy would need to be £102). |


| Jack is thinking of a 3-digit number. | 181 |
| :--- | :--- |
| When he multiplies his number by 100, | 262 |
| the ten thousands and hundreds digit are | 343 |
| the same. | 424 |
| The sum of the digits is 10 | 505 |
| What number could Jack be thinking of? |  |

## Divide by 10, 100 and 1,000

## Notes and Guidance

Children look at dividing by 10, 100 and 1,000 using a place value chart.

They use counters and digits to learn that the digits move to the right when dividing by powers of ten. They develop understanding of how many places to the right to move the counters to the right.

## Mathematical Talk

What happens to the digits?
How are dividing by 10,100 and 1,000 related to each other?
How are dividing by 10,100 and 1,000 linked to multiplying by 10,100 and 1,000 ?

What does 'inverse' mean?

## Varied Fluency

| HTh | TTh | Th | H | T | O |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | O | $\mathrm{O}_{\mathrm{O}} \mathrm{O}$ | O O |  |  |

What number is represented in the place value grid?
Divide the number by 100
Which direction do the counters move?
How many columns do they move? How do you know how many columns to move?
What number do we have now?
$\square$ Complete the following using a place value grid.

- Divide 460 by 10
- Divide 5,300 by 100
- Divide 62,000 by 1,000

Divide these numbers by 10,100 and 1,000
80,000 300,000

$$
547,000
$$

Calculate $45,000 \div 10 \div 10$
How else could you calculate this?

## Divide by 10, 100 and 1,000

## Reasoning and Problem Solving

| Mo has $£ 357,000$ in his bank. | $357,000 \div 1,000$ <br> $=357$ |
| :--- | :--- |
| He divides the amount by 1,000 and <br> takes that much money out of the bank. <br> If you subtract <br> Using the money he has taken out, he <br> buys some furniture costing two hundred <br> and sixty-nine pounds. | with $£ 88$ |
| How much money does Mo have left <br> from the money he took out? |  |
| Show your working out. |  |



Can you write at least two questions for each answer involving dividing by 10,100 or 1,000 ?

```
Possible
solutions:
3,970 \div10=397
57,000 \div 10=
5,700
397,000 \div 1,000
= 397
40,500 \div100=
4 0 5
620,300 \div 100 =
6,203
```


## Multiples of 10, 100 and 1,000

## Notes and Guidance

## Varied Fluency

Children have been taught how to multiply and divide by 10 , 100 and 1,000

They now use knowledge of other multiples of 10, 100 and 1,000 to answer related questions.

## Mathematical Talk

If we are multiplying by 20 , can we break it down into two steps and use our knowledge of multiplying by 10 ?

How does using multiplication and division as the inverse of the other help us to use known facts?
$36 \times 5=180$
Use this fact to solve the following questions:
$36 \times 50=$ $\qquad$ $500 \times 36=$ $\qquad$
$5 \times 360=$ $\qquad$

$$
360 \times 500=
$$

Here are two methods to solve $24 \times 20$

| Method 1 | Method 2 |
| :---: | :---: |
| $24 \times 10 \times 2$ |  |
| $=240 \times 2$ |  |
| $=480$ | $24 \times 2 \times 10$ <br> $=48 \times 10$ <br> $=480$ |

What is the same about the methods, what is different?
The division diagram shows $7,200 \div 200=36$

Use the diagram to solve:


$$
\begin{aligned}
& 3,600 \div 200= \\
& 18,000 \div 200= \\
& 5,400 \div \square=27 \\
& =6,600 \div 200
\end{aligned}
$$

## Multiples of 10, 100 and 1,000

## Reasoning and Problem Solving



$$
6 \times 7=42
$$

Alex uses this multiplication fact to solve

$$
420 \div 70=
$$

Alex says,


The answer is 60 because all of the numbers are 10 times bigger.

Do you agree with Alex?
Explain your answer.

Alex is wrong; both numbers (the dividend and divisor) are 10 times bigger than the numbers in the multiplication so the answer is 6 .
$6 \times 70=420$, therefore $420 \div$ $70=6$

