## White <br> Spring - Block 2 <br> R@se <br> Maths Fractions

## Overview

## Small Steps

## NC Objectives

Equivalent fractions

- Improper fractions to mixed numbers
- Mixed numbers to improper fractions
Number sequences
- Compare and order fractions less than 1
- Compare and order fractions greater than 1
- Add and subtract fractions
- Add fractions within 1
- Add 3 or more fractions
- Add fractions
- Add mixed numbers
Dubtract fractions
- Subtract mixed numbers
- Subtract - breaking the whole

Compare and order fractions whose denominators are multiples of the same number.

Identify, name and write equivalent fractions of a given fraction, represented visually including tenths and hundredths.

Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements $>1$ as a mixed number [for example $\left.\frac{2}{5}+\frac{4}{5}=\frac{6}{5}=1 \frac{1}{5}\right]$

Add and subtract fractions with the same denominator and denominators that are multiples of the same number.

## Overview

## Small Steps

## NC Objectives

Subtract 2 mixed numbers
Multiply unit fractions by an integer
Multiply non-unit fractions by an integer
Multiply mixed numbers by integers
Fraction of an amount
Using fractions as operators

Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

Read and write decimal numbers as
fractions [ for example $0.71=\frac{71}{100}$ ]
Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

## Equivalent Fractions

## Notes and Guidance

Children explore equivalent fractions using models and concrete representations.

They use models to make the link to multiplication and division. Children then apply the abstract method to find equivalent fractions.

It is important children have the conceptual understanding before moving on to just using an abstract method.

## Mathematical Talk

What equivalent fractions can we find by folding the paper? How can we record these?

What is the same and what is different about the numerators and denominators in the equivalent fractions?

How does multiplication and division help us find equivalent fractions? Where can we see this in our model?

## Varied Fluency

Take two pieces of paper the same size.

Fold one piece into two equal pieces. Fold the other into eight equal pieces. What equivalent fractions can you find?

Use the models to write equivalent fractions.


Eva uses the models and her multiplication and division skills to find equivalent fractions.


Eva uses the same approach to find equivalent fractions for these fractions. How will her method change?

$$
\frac{4}{12}=\frac{\square}{3} \quad \frac{6}{12}=\frac{\square}{4} \quad \frac{6}{12}=\frac{\square}{2}
$$

## Equivalent Fractions

## Reasoning and Problem Solving



Ron thinks you can only simplify even numbered fractions because you keep on halving the numerator and denominator until you get an odd number.

Do you agree?
Explain your answer.

Here are some fraction cards.
All of the fractions are equivalent.

$A+B=16$
Calculate the value of $C$.

Ron is wrong. For example $\frac{3}{9}$ can be simplified to $\frac{1}{3}$ and these are all odd numbers.
$A=10$
$B=6$
$C=15$

## Improper to Mixed Numbers

## Notes and Guidance

Children convert improper fractions to mixed numbers for the first time. An improper fraction is a fraction where the numerator is greater than the denominator. A mixed number is a number consisting of an integer and a proper fraction.

It is important for children to see this process represented visually to allow them to make the connections between the concept and what happens in the abstract.

## Mathematical Talk

How many parts are there in a whole?
What do you notice happens to the mixed number when the denominator increases and the numerator remains the same?

What happens when the numerator is a multiple of the denominator?

## Varied Fluency

Whitney converts the improper fraction $\frac{14}{5}$ into a mixed number using cubes.
She groups the cubes into 5 s , then has 4 left over.


Use Whitney's method to convert $\frac{11}{3}, \frac{11}{4}, \frac{11}{5}$ and $\frac{11}{6}$
$\square$ Tommy converts the improper fraction $\frac{27}{8}$ into a mixed number using bar models.


## Improper to Mixed Numbers

## Reasoning and Problem Solving



## Spot the mistake

- $\frac{27}{5}=5 \frac{1}{5}$
- $\frac{27}{3}=8$
- $\frac{27}{4}=5 \frac{7}{4}$
- $\frac{27}{10}=20 \frac{7}{10}$

What mistakes have been made?
Can you find the correct answers?

Correct answers

- $5 \frac{2}{5}$ (incorrect number of fifths)
- 9 (incorrect whole)
- $6 \frac{3}{4}$ (still have an improper
fraction)
- $2 \frac{7}{10}$ (incorrect number of wholes)


## Mixed Numbers to Improper

## Notes and Guidance

Children now convert from mixed numbers to improper fractions using concrete and pictorial methods to understand the abstract method.

Ensure children always write their working alongside the concrete and pictorial representations so they can see the clear links to the abstract.

## Mathematical Talk

How many quarters/halves/eighths/fifths are there in a whole?
How does multiplication support us in converting from mixed numbers to improper fractions?

Can you explain the steps in converting an improper fraction to a mixed number? Use the vocabulary: numerator, denominator, multiply, add

How could we use the previous bar model to help?

## Varied Fluency

Whitney converts $3 \frac{2}{5}$ into an improper fraction using cubes. 1 whole is equal to $\square$ fifths.

3 wholes are equal to $\square$ fifths.
$\square$ fifths + two fifths $=\square$ fifths
Use Whitney's method to convert $2 \frac{2}{3}, 2 \frac{2}{4}, 2 \frac{2}{5}$ and $2 \frac{2}{6}$
$\square$ Jack uses bar models to convert a mixed number into an improper fraction.

$2 \frac{3}{5}=\square$ wholes $+\square$ fifths


Use Jack's method to convert $2 \frac{1}{6}, 4 \frac{1}{6}, 4 \frac{1}{3}$ and $8 \frac{2}{3}$

## Mixed Numbers to Improper

## Reasoning and Problem Solving

\(\left.$$
\begin{array}{ll}\begin{array}{l}\text { Three children have incorrectly } \\
\text { converted } 3 \frac{2}{5} \text { into an improper fraction. }\end{array} & \begin{array}{l}\text { Annie has } \\
\text { multiplied the } \\
\text { numerator and } \\
\text { denominator by } 3\end{array}
$$ <br>

Mo has multiplied\end{array}\right\}\)| $3 \frac{2}{5}=\frac{6}{15}$ |
| :--- |
| the correctly but |
| then forgotten to |
| add on the extra 2 |
| parts. |

Fill in the missing numbers.
How many different possibilities can you find for each equation?

$$
\begin{aligned}
& 2 \frac{\square}{8}=\frac{\square}{8} \\
& 2 \frac{\square}{5}=\frac{\square}{5}
\end{aligned}
$$

Compare the number of possibilities you found.

$$
\begin{array}{ll}
2 \frac{1}{8}=\frac{17}{8} & 2 \frac{2}{8}=\frac{18}{8} \\
2 \frac{3}{8}=\frac{19}{8} & 2 \frac{4}{8}=\frac{20}{8} \\
2 \frac{5}{8}=\frac{21}{8} & 2 \frac{6}{8}=\frac{22}{8} \\
2 \frac{7}{8}=\frac{23}{8} & \\
\text { There will be 4 } \\
\text { solutions for fifths. }
\end{array}
$$

Teacher notes:
Encourage
children to make generalisations that the number of solutions is one less than the denominator.

## Number Sequences

## Notes and Guidance

## Varied Fluency

Children count up and down in a given fraction. They continue to use visual representations to help them explore number sequences.

Children also find missing fractions in a sequence and determine whether the sequence is increasing or decreasing and by how much.

## Mathematical Talk

What are the intervals between the fractions?

Are the fractions increasing or decreasing? How much are they increasing or decreasing by?

Can you convert the mixed numbers to improper fractions? Does this make it easier to continue the sequence?

Use the counting stick to count up and down in these fractions.

- Start at 0 and count up in steps of $\frac{1}{4}$
- Start at 4 and count down in steps of $\frac{1}{3}$
- Start at 1 and count up in steps of $\frac{2}{3}$
$\square$ Complete the missing values on the number line.

$\square$ Complete the sequences.
$\frac{3}{4}, \square, 1 \frac{3}{4}, 2 \frac{1}{4}$
$\frac{\square}{\square}, 3 \frac{1}{3}, \frac{\square}{\square}, 2 \frac{2}{3}$
$\frac{\square}{\square}, 5 \frac{1}{2}, 5 \frac{7}{10}, 5 \frac{9}{10}$
$\frac{3}{5}, \frac{\square}{\square}, \frac{\square}{\square}, 3$


## Number Sequences

## Reasoning and Problem Solving

| Three children are counting in quarters. | They are all <br> correct, they are <br> all counting in <br> quarter. |
| :--- | :--- |
| Teddy has |  |
| simplified all |  |
| answers and Eva |  |
| has converted |  |
| improper fractions |  |
| to mixed numbers. |  |

Play the fraction game for four players. Place the four fraction cards on the floor. Each player stands in front of a fraction. We are going to count up in tenths starting at 0
When you say a fraction, place your foot on your fraction.


How can we make 4 tenths?
What is the highest fraction we can count to?
How about if we used two feet?

Children can make four tenths by stepping on one tenth and three tenths at the same time.
With one foot, they can count up to 11 tenths or one and one tenth.
With two feet they can count up to 22 tenths.

## Compare \& Order (Less than 1)

## Notes and Guidance

Children build on their equivalent fraction knowledge to compare and order fractions less than 1 where the denominators are multiples of the same number.

Children compare the fractions by finding a common denominator or a common numerator. They use bar models to support their understanding.

## Mathematical Talk

How does a bar model help us to visualise the fractions? Should both of our bars be the same size? Why? What does this show us?
If the numerators are the same, how can we compare our fractions?
If the denominators are the same, how can we compare our fractions?
Do we always have to find a common denominator? Can we find a common numerator?

## Varied Fluency

Use bar models to compare $\frac{5}{8}$ and $\frac{3}{4}$


Use this method to help you compare:
$\frac{5}{6}$ and $\frac{2}{3} \quad \frac{2}{3}$ and $\frac{5}{9} \quad \frac{7}{16}$ and $\frac{3}{8}$
U Use common numerators to help you compare $\frac{2}{5}$ and $\frac{2}{3}$


Use this method to help you compare:
$\frac{6}{7}$ and $\frac{6}{8} \quad \frac{4}{9}$ and $\frac{4}{5} \quad \frac{4}{11}$ and $\frac{2}{5}$
$\square$ Order the fractions from greatest to smallest: $\frac{3}{7}, \frac{3}{5}$ and $\frac{3}{8} \quad \frac{2}{3}, \frac{5}{6}$ and $\frac{7}{12} \quad \frac{6}{11}, \frac{3}{5}$ and $\frac{2}{3}$

## Compare \& Order (Less than 1)

## Reasoning and Problem Solving



## Always, sometimes, never?

If one denominator is a multiple of the other you can simplify the fraction with the larger denominator to make the denominators the same.

Example:
Could $\frac{?}{4}$ and $\frac{?}{12}$ be simplified to $\frac{?}{4}$ and $\frac{?}{4}$ ?
Prove it.

## Sometimes

It does not work
for some fractions
e.g. $\frac{8}{15}$ and $\frac{3}{5}$

But does work for others e.g. $\frac{1}{4}$ and $\frac{9}{12}$

## Compare \& Order (More than 1)

## Notes and Guidance

Children use their knowledge of ordering fractions less than 1 to help them compare and order fractions greater than 1

They use their knowledge of common denominators to help them.

Children will compare both improper fractions and mixed numbers during this step.

## Mathematical Talk

How can we represent the fractions?
How does the bar help us see which fraction is the greatest?
Can we use our knowledge of multiples to help us?
Can you predict which fractions will be greatest? Explain how you know.

Is it more efficient to compare using numerators or denominators?

## Varied Fluency

Use bar models to compare $\frac{7}{6}$ and $\frac{5}{3}$


Use this method to help you compare:
$\frac{5}{2}$ and $\frac{9}{4} \quad \frac{11}{6}$ and $\frac{5}{3} \quad \frac{9}{4}$ and $\frac{17}{8}$
$\square$ Use a bar model to compare $1 \frac{2}{3}$ and $1 \frac{5}{6}$


Use this method to help you compare:
$1 \frac{3}{4}$ and $1 \frac{3}{8} \quad 1 \frac{5}{8}$ and $1 \frac{1}{2} \quad 2 \frac{3}{7}$ and $2 \frac{9}{14}$
$\square$ Order the fractions from
greatest to smallest
using common
denominators:


## Compare \& Order (More than 1)

## Reasoning and Problem Solving



Dora looks at the fractions $1 \frac{7}{12}$ and $1 \frac{3}{4}$
She says,


Do you agree?
Explain why using a model.

Possible answer: I do not agree because $1 \frac{3}{4}$ is equivalent to $1 \frac{9}{12}$ and this is greater than $1 \frac{7}{12}$

## Add \& Subtract Fractions

## Notes and Guidance

Children recap their Year 4 understanding of adding and subtracting fractions with the same denominator.

They use bar models to support understanding of adding and subtracting fractions.

## Mathematical Talk

How many equal parts do I need to split my bar into?
Can you convert the improper fraction into a mixed number?
How can a bar model help you balance both sides of the equals sign?

## Varied Fluency

Here is a bar model to calculate $\frac{3}{5}+\frac{4}{5}$


Use a bar model to solve the calculations:

$$
\frac{3}{8}+\frac{3}{8} \quad \frac{5}{6}+\frac{1}{6} \quad \frac{5}{3}+\frac{5}{3}
$$

$\square$ Here are two bar models to calculate $\frac{7}{8}-\frac{3}{8}$
$\square$


What is the difference between the two methods?
Use your preferred method to calculate:
$\frac{5}{8}-\frac{1}{8} \quad \frac{9}{7}-\frac{4}{7} \quad \frac{5}{3}-\frac{5}{3} \quad 1-\frac{2}{5}$
$\square$ Calculate:
$\frac{3}{7}+\frac{5}{7}=\frac{\square}{\square}+\frac{4}{7} \quad \frac{9}{5}-\frac{5}{5}=\frac{6}{5}-\frac{\square}{\square} \quad \frac{2}{3}+\frac{\square}{\square}=\frac{11}{3}-\frac{4}{3}$

## Add \& Subtract Fractions

## Reasoning and Problem Solving

How many different ways can you
balance the equation?

$$
\frac{5}{9}+\frac{\square}{9}=\frac{8}{9}+\frac{\square}{9}
$$

Possible answers:

$$
\frac{5}{9}+\frac{3}{9}=\frac{8}{9}+\frac{0}{9}
$$

$$
\frac{5}{9}+\frac{4}{9}=\frac{8}{9}+\frac{1}{9}
$$

$$
\frac{5}{9}+\frac{5}{9}=\frac{8}{9}+\frac{2}{9}
$$

Any combination of fractions where the numerators add up to the same total on each side of the equals sign.

A chocolate bar has 12 equal pieces.
Amir eats $\frac{5}{12}$ more of the bar than Whitney.

There is one twelfth of the bar remaining.
What fraction of the bar does Amir eat?
What fraction of the bar does Whitney eat?

Amir eats $\frac{8}{12}$ of the chocolate bar and Whitney eats $\frac{3}{12}$ of the chocolate bar.

## Add Fractions within 1

## Notes and Guidance

## Varied Fluency

Children add fractions with different denominators for the first time where one denominator is a multiple of the other.

They use pictorial representations to convert the fractions so they have the same denominator.

Ensure children always write their working alongside the pictorial representations so they see the clear links.

## Mathematical Talk

Can you find a common denominator? Do you need to convert both fractions or just one?

Can you explain Mo and Rosie's methods to a partner? Which method do you prefer?

How do Mo and Rosie's methods support finding a common denominator?

Mo is calculating $\frac{1}{2}+\frac{1}{8}$
He uses a diagram to represent the sum.

$\frac{1}{2}+\frac{1}{8}=\frac{4}{8}+\frac{1}{8}=\frac{5}{8}$

Use Mo's method to solve :

$$
\frac{1}{2}+\frac{3}{8} \quad \frac{1}{4}+\frac{3}{8} \quad \frac{7}{10}+\frac{1}{5}
$$

$\square$ Rosie is using a bar model to solve $\frac{1}{4}+\frac{3}{8}$

$\frac{1}{4}+\frac{3}{8}=\frac{2}{8}+\frac{3}{8}=\frac{5}{8}$


Use a bar model to solve:

$$
\frac{1}{6}+\frac{5}{12} \quad \frac{2}{9}+\frac{1}{3} \quad \frac{1}{3}+\frac{4}{15}
$$

## Add Fractions within 1

## Reasoning and Problem Solving

| $\begin{aligned} & \frac{5}{16}+\frac{\square}{8}=\frac{15}{16} \\ & \frac{\square}{20}+\frac{7}{10}=\frac{17}{20} \end{aligned}$ |  |
| :---: | :---: |
| Annie solved this calculation. $\begin{aligned} \frac{3}{4}+\frac{3}{16} & =\frac{3+3}{4+16} \\ & =\frac{6}{20} \\ & =\frac{3}{10} \end{aligned}$ <br> Can you spot and explain her mistake? | Annie is wrong because she has just added the numerators and the denominators. When adding fractions with different denominators you need to find a common denominator. |

Two children are solving $\frac{1}{3}+\frac{4}{15}$
Eva starts by drawing this model:


Alex starts by drawing this model:


Can you explain each person's method and how they would complete the question?
Which method do you prefer and why?

Possible answer:
Each child may
have started with a different fraction in the calculation. e.g. Eva has started by shading a third. She now needs to divide each third into five equal parts so
there are fifteen equal parts
altogether. Eva will then shade $\frac{4}{15}$ and will have
$\frac{9}{15}$ altogether.

## Add 3 or More Fractions

## Notes and Guidance

Children add more than 2 fractions where two denominators are a multiple of the other.

They use a bar model to continue exploring this.
Ensure children always write their working alongside the pictorial representations so they see the clear links.

## Mathematical Talk

Can you find a common denominator? Do you need to convert both fractions or just one?

Can you explain Ron's method to a partner? How does Ron's method support finding a common denominator?

Can you draw what Farmer Staneff's field could look like? What fractions could you divide your field into?

Why would a bar model not be efficient for this question?

## Varied Fluency

Ron uses a bar model to calculate $\frac{2}{5}+\frac{1}{10}+\frac{3}{20}$

$\frac{4}{10} \quad \frac{1}{10}$
Use a bar model to solve:
$\frac{1}{4}+\frac{3}{8}+\frac{5}{16} \quad \frac{1}{2}+\frac{1}{6}+\frac{1}{12}$
$\square$ Farmer Staneff owns a field.
He plants carrots on $\frac{1}{3}$ of the field.
He plants potatoes on $\frac{2}{9}$ of the field.
He plants onions on $\frac{5}{18}$ of the field.
What fraction of the field is covered altogether?
$\square$ Complete the fractions.

$$
\frac{1}{5}+\frac{\square}{10}+\frac{8}{20}=1 \quad \frac{1}{5}+\frac{\square}{15}+\frac{1}{30}=1
$$

## Add 3 or More Fractions

## Reasoning and Problem Solving

| Eva is attempting to answer: | Eva is wrong <br> because she has <br> added the <br> numerators and <br> denominators <br> together and <br> hasn't found a <br> common <br> denominator. The <br> correct answer is <br> $\frac{17}{20}$ |
| :--- | :--- |
| Do you agree with Eva? |  |
| Explain why. |  |

Jack has added 3 fractions together to get an answer of $\frac{17}{18}$

| -1 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

What 3 fractions could he have added?
Can you find more than one answer?

Possible answers:

$$
\frac{1}{18}+\frac{4}{18}+\frac{13}{18}
$$

$$
\frac{1}{9}+\frac{5}{9}+\frac{5}{18}
$$

$$
\frac{1}{6}+\frac{5}{9}+\frac{2}{9}
$$

$$
\frac{1}{18}+\frac{1}{6}+\frac{13}{18}
$$

$$
\frac{1}{3}+\frac{1}{6}+\frac{4}{9}
$$

## Add Fractions

## Notes and Guidance

Children continue to represent adding fractions using pictorial methods to explore adding two or more proper fractions where the total is greater than 1

Children can record their totals as an improper fraction but will then convert this to a mixed number using their prior knowledge.

## Mathematical Talk

How does the pictorial method support me to add the fractions?
Which common denominator will we use?
How do my times-tables support me to add fractions?
Which representation do you prefer? Why?

## Varied Fluency


$\frac{1}{3}+\frac{5}{6}+\frac{5}{12}=1 \frac{7}{12}$
Explain each step of the calculation.
Use this method to help you add the fractions.
Give your answer as a mixed number.
$\frac{2}{3}+\frac{1}{6}+\frac{7}{12} \quad \frac{1}{4}+\frac{7}{8}+\frac{3}{16} \quad \frac{1}{2}+\frac{5}{6}+\frac{5}{12}$
$\square$ Use the bar model to add the fractions. Record your answer as a mixed number.
$\frac{3}{4}+\frac{3}{8}+\frac{1}{2}=$


Draw your own models to solve:

$$
\frac{5}{12}+\frac{1}{6}+\frac{1}{2} \quad \frac{11}{20}+\frac{3}{5}+\frac{1}{10} \quad \frac{3}{4}+\frac{5}{12}+\frac{1}{2}
$$

## Add Fractions

## Reasoning and Problem Solving

Annie is adding three fractions. She uses the model to help her.


What could her three fractions be?

How many different combinations can you find?

Can you write a number story to represent your calculation?

## Possible answer:

$\frac{2}{3}+\frac{4}{12}+\frac{1}{2}=1 \frac{1}{2}$
Other equivalent fractions may be used.

Example story: Some children are eating pizzas. Jack eats two thirds, Amir eats four twelfths and
Dexter eats half a pizza. How much pizza did they eat altogether?

The sum of three fractions is $2 \frac{1}{8}$
The fractions have different denominators.

All of the fractions are greater than or equal to a half.

None of the fractions are improper fractions.

All of the denominators are factors of 8
What could the fractions be?

$$
\frac{1}{2}+\frac{3}{4}+\frac{7}{8}
$$

Children could be given less clues and explore other possible solutions.

## Add Mixed Numbers

## Notes and Guidance

## Varied Fluency

Children move on to adding two fractions where one or both are mixed numbers or improper fractions.

They will use a method of adding the wholes and then adding the parts. Children will record their answer in its simplest form.

Children can still draw models to represent adding fractions.

## Mathematical Talk

How can we partition these mixed numbers into whole numbers and fractions?

What will the wholes total? Can I add the fractions straight away?

What will these mixed numbers be as improper fractions?
If I have an improper fraction in the question, should I change it to a mixed number first? Why?

$$
\begin{aligned}
& 1+2=3 \\
& \frac{1}{3}+\frac{1}{6}=\frac{2}{6}+\frac{1}{6}=\frac{3}{6}
\end{aligned}
$$

Add the fractions by adding the whole first and then the fractions.
Give your answer in its simplest form.

$$
3 \frac{1}{4}+2 \frac{3}{8} \quad 4 \frac{1}{9}+3 \frac{2}{3} \quad 2 \frac{5}{12}+2 \frac{1}{3}
$$

$\square$

$$
1 \frac{3}{4}+2 \frac{1}{8}=\frac{7}{4}+\frac{17}{8}=\frac{14}{8}+\frac{17}{8}=\frac{31}{8}=3 \frac{7}{8}
$$

Add the fractions by converting them to improper fractions.
$1 \frac{1}{4}+2 \frac{5}{12}$
$2 \frac{1}{9}+1 \frac{1}{3}$
$2 \frac{1}{6}+2 \frac{2}{3}$
$\square$ Add these fractions.
$4 \frac{7}{9}+2 \frac{1}{3}$
$\frac{17}{6}+1 \frac{1}{3}$
$\frac{15}{8}+2 \frac{1}{4}$

How do they differ from previous examples?

## Add Mixed Numbers

## Reasoning and Problem Solving

| Jack and Whitney have some juice. | They drink <br> $4 \frac{2}{3}$ litres <br> altogether. |
| :--- | :--- |
| Jack drinks $2 \frac{1}{4}$ litres and Whitney drinks <br> $2 \frac{5}{12}$ litres. | Encourage <br> children to justify <br> which method <br> they prefer and <br> why. Ensure <br> children discuss <br> which method is <br> more or less <br> efficient. |
| Complete this using two different <br> methods. | Which method do you think is more <br> efficient? Why? |

$$
\text { Fill in the missing numbers. } 5 \frac{3}{6} \text { or } 5 \frac{1}{2}
$$

## Subtract Fractions

## Notes and Guidance

Children subtract fractions with different denominators for the first time, where one denominator is a multiple of the other.

It is important that subtraction is explored as both take away and finding the difference.

## Mathematical Talk

What could the common denominator be?
Can you draw a model to help you solve the problem?
Is it easier to use a take away bar model (single bar model) or a bar model to find the difference (comparison model)?

## Varied Fluency



Explain each step of the calculation.
Use this method to help you solve $\frac{5}{6}-\frac{1}{3}$ and $\frac{7}{8}-\frac{5}{16}$
Tommy and Teddy both have the same sized chocolate bar. Tommy has $\frac{3}{4}$ left, Teddy has $\frac{5}{12}$ left.


How much more does Tommy have?
$\square$ Amir uses a number line to find the difference between $\frac{5}{9}$ and $\frac{4}{3}$


Use this method to find the difference between:

$$
\frac{3}{4} \text { and } \frac{5}{12} \quad \frac{19}{15} \text { and } \frac{3}{5} \quad \frac{20}{9} \text { and } \frac{4}{3}
$$

## Subtract Fractions

## Reasoning and Problem Solving

| Which subtraction is the odd one out? |  | Possible answers: <br> $C$ is the odd one out because the denominators aren't multiples of each other. |
| :---: | :---: | :---: |
| A | $\frac{13}{4}-\frac{3}{8}$ |  |
| $B$ | $\frac{10}{3}-\frac{2}{9}$ | each other. <br> $A$ is the odd one |
| C | $\frac{23}{7}-\frac{1}{3}$ | denominators are even. |
| Expl |  | out because it is the only answer above 3 |


| The perimeter of the rectangle is $\frac{16}{9}$ | The missing |
| :---: | :--- |
| 2 | length is $\frac{2}{9}$ |

## Subtract Mixed Numbers (1)

## Notes and Guidance

Children apply their understanding of subtracting fractions where one denominator is a multiple of the other to subtract proper fractions from mixed numbers.

They continue to use models and number lines to support their understanding.

## Mathematical Talk

Which fraction is the greatest? How do you know?
If the denominators are different, what can we do?
Can you simplify your answer?
Which method do you prefer when subtracting fractions: taking away or finding the difference?

## Varied Fluency



$$
1 \frac{3}{4}-\frac{5}{8}=1 \frac{1}{8}
$$

Use this method to help you solve:
$2 \frac{3}{5}-\frac{3}{10}$
$1 \frac{2}{3}-\frac{1}{6}$
$1 \frac{5}{6}-\frac{7}{12}$
$\square$ Use a number line to find the difference between $1 \frac{2}{5}$ and $\frac{3}{10}$

$$
1 \frac{2}{5}=1 \frac{4}{10}
$$



Use a number line to find the difference between:

$$
3 \frac{5}{6} \text { and } \frac{1}{12} \quad 5 \frac{5}{7} \text { and } \frac{3}{14} \quad 2 \frac{7}{9} \text { and } \frac{11}{18}
$$

Solve:
$1 \frac{2}{3}-\frac{5}{6}$
$1 \frac{3}{4}-\frac{7}{8}$
$2 \frac{3}{8}-\frac{11}{16}$

## Subtract Mixed Numbers (1)

## Reasoning and Problem Solving

| Amir is attempting to solve $2 \frac{5}{14}-\frac{2}{7}$ | Possible answer: <br> Here is his working out: |
| :--- | :--- |
| Amir is wrong <br> because he hasn't <br> found a common <br> denominator when <br> subtracting the <br> fractions he has <br> just subtracted the <br> numerators and <br> the denominators. <br> The correct <br> answer is $2 \frac{1}{14}$ |  |
| Explain your answer. | a $\frac{5}{14}-\frac{2}{7}=2 \frac{3}{7}$ |

Here is Rosie's method. What is the calculation?


Can you find more than one answer? Why is there more than one answer?

The calculation could be $1 \frac{5}{6}-\frac{7}{12}$ or $1 \frac{10}{12}-\frac{7}{12}$

There is more than one answer because five sixths and ten twelfths are equivalent.
Children should be encouraged to write the question as $1 \frac{5}{6}-\frac{7}{12}$ so that all fractions are in their simplest form.

## Subtract Mixed Numbers (2)

## Notes and Guidance

Children use prior knowledge of fractions to subtract two fractions where one is a mixed number and you need to break one of the wholes up.

They use the method of flexible partitioning to create a new mixed number so they can complete the calculation.

## Mathematical Talk

Is flexible partitioning easier than converting the mixed number to an improper fraction?

Do we always have to partition the mixed number?
When can we subtract a fraction without partitioning the mixed number in a different way?

## Varied Fluency

We can work out $2 \frac{3}{4}-\frac{7}{8}$ using this method.


Use this method to calculate:
$3 \frac{1}{3}-\frac{5}{6}$
$4 \frac{1}{5}-\frac{7}{10}$
$5 \frac{2}{3}-\frac{4}{9}$
$\square$ Use flexible partitioning to solve $7 \frac{1}{3}-\frac{5}{6}$

$$
7 \frac{1}{3}-\frac{5}{6}=6+1 \frac{1}{3}-\frac{5}{6}=6+1 \frac{2}{6}-\frac{5}{6}=6 \frac{3}{6}=6 \frac{1}{2}
$$

Use this method to calculate:
$4 \frac{2}{3}-\frac{5}{6}$
$4 \frac{1}{5}-\frac{7}{15}$
$5 \frac{1}{4}-\frac{7}{8}$
$\square$ Mr Brown has $3 \frac{1}{4}$ bags of flour. He uses $\frac{7}{8}$ of a bag. How much flour does he have left?

## Subtract Mixed Numbers (2)

## Reasoning and Problem Solving

Place 2,3 and 4 in the boxes to make
the calculation correct.

$$
27 \frac{1}{\square}-\frac{\square}{3}=26 \frac{1}{3}=26 \frac{2}{3}
$$

3 children are working out $6 \frac{2}{3}-\frac{5}{6}$
They partition the mixed number in the following ways to help them.


## Subtract 2 Mixed Numbers

## Notes and Guidance

Children use different strategies to subtract two mixed numbers.

Building on learning in previous steps, they look at partitioning the mixed numbers into wholes and parts and build on their understanding of flexible partitioning as well as converting to improper fractions when an exchange is involved.

## Mathematical Talk

Why is subtracting the wholes and parts separately easier with some fractions than others?

Can you show the subtraction as a difference on a number line? Bar model? How are these different to taking away?

Does making the whole numbers larger make the subtraction any more difficult? Explain why.

## Varied Fluency

Here is a bar model to calculate $3 \frac{5}{8}-2 \frac{1}{4}$

| $\square$ |
| :--- |
| $\square$ |
| $3-2=1$ |

$$
\begin{aligned}
& \triangle X|||l| \\
& \frac{5}{8}-\frac{1}{4}=\frac{5}{8}-\frac{2}{8}=\frac{3}{8}
\end{aligned} \quad 3 \frac{5}{8}-2 \frac{1}{4}=1 \frac{3}{8}
$$

Use this method to calculate:
$3 \frac{7}{8}-2 \frac{3}{4}$
$5 \frac{5}{6}-2 \frac{1}{3}$
$3 \frac{8}{9}-2 \frac{5}{27}$

Why does this method not work effectively for $5 \frac{1}{6}-2 \frac{1}{3}$ ?
$\square$ Here is a method to calculate $5 \frac{1}{6}-2 \frac{1}{3}$

$5 \frac{1}{6}-2 \frac{1}{3}=4 \frac{7}{6}-2 \frac{1}{3}=4 \frac{7}{6}-2 \frac{2}{6}=2 \frac{5}{6}$

> Use this method to calculate: $\begin{array}{lll}3 \frac{1}{4}-2 \frac{5}{8} & 5 \frac{1}{3}-2 \frac{7}{12} & 27 \frac{1}{3}-14 \frac{7}{15}\end{array}$

## Subtract 2 Mixed Numbers

## Reasoning and Problem Solving

| There are three colours of dog biscuits in <br> a bag of dog food: red, brown and <br> orange. | $3 \frac{3}{4}+1 \frac{7}{16}=5 \frac{3}{16}$ |
| :--- | :--- |
| The total mass of the dog food is 7 kg. | $7-5 \frac{3}{16}=1 \frac{13}{16}$ |
| The mass of |  |
| The mass of red biscuits is $3 \frac{3}{4} \mathrm{~kg}$ and the |  |
| mass of the brown biscuits is $1 \frac{7}{16} \mathrm{~kg}$. | $1 \frac{13}{16} \mathrm{~kg}$. | | What is the mass of orange biscuits? |
| :--- |


| Rosie has $20 \frac{3}{4} \mathrm{~cm}$ of ribbon. | Annie has $13 \frac{7}{8} \mathrm{~cm}$ <br> of ribbon. |
| :--- | :--- |
| Annie has $6 \frac{7}{8} \mathrm{~cm}$ less ribbon than Rosie. | Altogether they <br> have $34 \frac{5}{8} \mathrm{~cm}$ of <br> How much ribbon does Annie have? |
| How much ribbon do they have <br> altogether? | ribbon. |

## Multiply by an Integer (1)

## Notes and Guidance

Children are introduced to multiplying fractions by a whole number for the first time. They link this to repeated addition and see that the denominator remains the same, whilst the numerator is multiplied by the integer.
This is shown clearly through the range of models to build the children's conceptual understanding of multiplying fractions.
Children should be encouraged to simplify fractions where possible.

## Mathematical Talk

How is multiplying fractions similar to adding fractions?
What is the same/different between: $\frac{3}{4} \times 2$ and $2 \times \frac{3}{4}$ ?
Which bar model do you find the most useful?
Which bar model helps us to convert from an improper fraction to a mixed number most effectively?

What has happened to the numerator/denominator?

## Varied Fluency

Work out $\frac{1}{6} \times 4$ by counting in sixths.
$\frac{1}{6} \times 4=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$
Use this method to work out:
$2 \times \frac{1}{3}$
$\frac{1}{5} \times 3$


4 sixthsMo uses a single bar model to work out: $\frac{1}{5} \times 4=\frac{4}{5}$


Use this method to work out:
$\frac{1}{4} \times 3$
$6 \times \frac{1}{8}$
$\frac{1}{10} \times 8$
$\square$ Eva uses a number line and repeated addition to work out
$\frac{1}{5} \times 7=\frac{7}{5}=1 \frac{2}{5}$
Use this method to work out:
$5 \times \frac{1}{8}$
$\frac{1}{3} \times 3$

$\frac{1}{4} \times 7$

## Multiply by an Integer (1)

## Reasoning and Problem Solving

| Amir is multiplying fractions by a whole <br> number. | Amir has <br> multiplied both the <br> numerator and the <br> denominator so he <br> has found an <br> equivalent fraction. <br> Encourage <br> children to draw <br> models to <br> represent this <br> correctly. |
| :--- | :--- |
| Can you explain his mistake? | Always - because <br> the numerator was <br> 1 it will always be <br> the same as your <br> denominator when <br> multiplied which <br> means that it is a <br> whole. |
| e.g. $\frac{1}{3} \times 3=\frac{3}{3}=1$ |  |

Amir is multiplying fractions by a whole number.

$$
\frac{1}{5} \times 5=\frac{5}{25}
$$

Can you explain his mistake?

## Always, sometimes, never?

When you multiply a unit fraction by the same number as it's denominator the answer will be one whole.

I am thinking of a unit fraction.
When I multiply it by 4 it will be equivalent to $\frac{1}{2}$

When I multiply it by 2 it will be equivalent to $\frac{1}{4}$

What is my fraction?
What do I need to multiply my fraction by so that my answer is equivalent to $\frac{3}{4}$ ?
$\frac{1}{8}$ because
$4 \times \frac{1}{8}=\frac{4}{8}=\frac{1}{2}$
and
$2 \times \frac{1}{8}=\frac{2}{8}=\frac{1}{4}$

Can you create your own version of this problem?

6 because
$6 \times \frac{1}{8}=\frac{6}{8}=\frac{3}{4}$

## Multiply by an Integer (2)

## Notes and Guidance

Children apply prior knowledge of multiplying a unit fraction by a whole number to multiplying a non-unit fraction by a whole number.
They use similar models and discuss which method will be the most efficient depending on the questions asked. Reinforce the concept of commutativity by showing examples of the fraction first and the integer first in the multiplication.

## Mathematical Talk

Can you show me 3 lots of $\frac{3}{10}$ on a bar model?
How many tenths do we have altogether?
How does repeated addition help us with this multiplication?
How does a number line help us see the multiplication?

## Varied Fluency

$\square$ Count the number of ninths to work $3 \times \frac{2}{9}$

| ! ! ! | - |  |  |
| :---: | :---: | :---: | :---: |
| ! $\%$ | $\square$ | $\square$ |  |
|  |  |  |  |

Use this method to work out:
$\frac{3}{8} \times 2$
$\frac{5}{16} \times 3$
$4 \times \frac{2}{11}$
$\square$ Use the model to help you solve $3 \times \frac{2}{10}$


Use this method to work out:
$\frac{2}{7} \times 3$
$\frac{3}{16} \times 4$
$4 \times \frac{5}{12}$
$\square$ Use the number line to help you solve $2 \times \frac{3}{7}$

Use this method to work out:

$\frac{3}{10} \times 3$
$\frac{2}{7} \times 2$
$4 \times \frac{3}{20}$

## Multiply by an Integer (2)

## Reasoning and Problem Solving




## Multiply by an Integer (3)

## Varied Fluency

Use repeated addition to work out $2 \frac{2}{3} \times 4$

$$
2 \frac{2}{3} \times 4=2 \frac{2}{3}+2 \frac{2}{3}+2 \frac{2}{3}+2 \frac{2}{3}=8 \frac{8}{3}=10 \frac{2}{3}
$$

Use this method to solve:

$$
2 \frac{1}{6} \times 3 \quad 1 \frac{3}{7} \times 2 \quad 3 \frac{1}{3} \times 4
$$

$\square$ Partition your fraction to help you solve $2 \frac{3}{4} \times 3$

$$
\begin{aligned}
& \square \square \square \square 2 \times 3=6 \\
& \square \square \square \square \frac{3}{4} \times 3=\frac{9}{4}=2 \frac{1}{4} \\
& \square \square \square \square+2 \frac{1}{4}=8 \frac{1}{4}
\end{aligned}
$$

Use this method to answer:
$2 \frac{5}{6} \times 3 \quad 3 \frac{4}{7} \times 2 \quad 2 \frac{1}{3} \times 5$

$$
1 \frac{5}{6} \times 3=\frac{11}{6} \times 3=\frac{33}{6}=5 \frac{3}{6}=5 \frac{1}{2}
$$

$\square$ Convert to an improper fraction to calculate:

$$
3 \frac{2}{7} \times 4 \quad 2 \frac{4}{9} \times 2 \quad 4 \times 3 \frac{3}{5}
$$

## Notes and Guidance

Children use their knowledge of fractions to multiply a mixed number by a whole number.

They use the method of repeated addition, multiplying the whole and part separately and the method of converting to an improper fraction then multiplying.

Continue to explore visual representations such as the bar model.

## Mathematical Talk

How could you represent this mixed number?
What is the denominator? How do you know?
How many wholes are there? How many parts are there?
What is multiplying fractions similar to? (repeated addition)
What representation could you use to convert a mixed number to an improper fraction?
(

## Multiply by an Integer (3)

## Reasoning and Problem Solving

| Jack runs $2 \frac{2}{3}$ miles three times per week. | Jack runs $2 \frac{2}{3} \times$ <br> $3=8$ miles. |
| :--- | :--- |
| Dexter runs $3 \frac{3}{4}$ miles twice a week. | Dexter runs <br> Who runs the furthest during the week? <br> $3 \frac{3}{4} \times 2=7 \frac{1}{2}$ <br> miles. <br> Explain your answer. <br> Jack runs further <br> by half a mile. |
|  |  |



## Fraction of an Amount

## Notes and Guidance

Children recap previous learning surrounding finding unit and non-unit fractions of amounts, quantities and measures.

It is important that the concept is explored pictorially through bar models to support children to make sense of the abstract.

## Mathematical Talk

How many equal groups have you shared 49 into? Why?
What does each equal part represent as a fraction and an amount?

What could you do to 1 metre to make the calculation easier?
1 litre = $\qquad$ $\mathrm{ml} 1 \mathrm{~kg}=$ $\qquad$ g

## Varied Fluency

$\square$ Find $\frac{1}{7}$ of 42


$$
\begin{aligned}
& 42 \div 7=6 \\
& \frac{1}{7} \text { of } 42 \text { is } 6
\end{aligned}
$$

Use this method to find:
$\frac{1}{8}$ of 56

$$
\frac{1}{6} \text { of } 480
$$

$\frac{1}{9}$ of 81 m
$\square$ Find $\frac{2}{7}$ of 42


$$
\begin{aligned}
& 42 \div 7=6 \\
& 6 \times 2=12 \\
& \frac{2}{7} \text { of } 42 \text { is } 12
\end{aligned}
$$

Use this method to find:
$\frac{3}{8}$ of $56 \quad \frac{5}{6}$ of $480 \quad \frac{4}{9}$ of 81 m
$\square$ Draw a bar model to help you calculate:
$\frac{4}{5}$ of $1 \mathrm{~m} \quad \frac{5}{12}$ of 1.44 litres $\quad \frac{3}{7}$ of 21 kg

## Fraction of an Amount

## Reasoning and Problem Solving

| Write a problem that matches the bar |
| :--- |
| model. |


| What other questions could you ask from |
| :--- | :--- |
| this model? | | Possible response: |
| :--- |
| There are 96 cars <br> in a car park. <br> $\frac{3}{8}$ of them are red. <br> How many cars <br> are red? |
| How many were <br> not red? etc. |
| There are 18 girls in the class. |
| How many children are in the class? |



## Fractions as Operators

## Notes and Guidance

Children link their understanding of fractions of amounts and multiplying fractions to use fractions as operators.

They use their knowledge of commutativity to help them understand that you can change the order of multiplication without changing the product.

## Mathematical Talk



What is the same and different about these bar models?

Is it easier to multiply a fraction or find a fraction of an amount? Does it depend on the whole number you are multiplying by? Can you see the link between the numbers?

## Varied Fluency

Tommy has calculated and drawn a bar model for two calculations.


What's the same and what's different about Tommy's calculations?
$\square$ Complete:
2 lots of $\frac{1}{10}=\square \quad \frac{1}{10}$ of $2=\square$
6 lots of $\square=3 \quad \square$ of $6=3$
Which calculation on each row is easier? Why?
8 lots of $\frac{1}{4}=\square \quad \frac{1}{4}$ of $8=\square$
Use this to complete:
$20 \times \frac{4}{5}=\frac{\square}{\square}$ of $20=\square$
$\square \times \frac{2}{3}=\frac{\square}{\square}$ of $18=12$
$\square \times \frac{1}{3}=\frac{1}{3}$ of $\square=20$

## Fractions as Operators

## Reasoning and Problem Solving

Which method would you use to complete these calculations: multiply the fractions or find the fraction of an amount?

Explain your choice for each one. Compare your method to your partner.

$$
\begin{aligned}
& 25 \times \frac{3}{5} \text { or } \frac{3}{5} \text { of } 25 \\
& 6 \times \frac{2}{3} \text { or } \frac{2}{3} \text { of } 6 \\
& 5 \times \frac{3}{8} \text { or } \frac{3}{8} \text { of } 5
\end{aligned}
$$

Possible response:

1. Children may find it easier to find 3 fifths of 25 rather than multiply 25 by 3
2. Children may choose either as they are of similar efficiency.
3. Children will probably find it more efficient to multiply than divide 5 by 8

Dexter and Jack are thinking of a twodigit number between 20 and 30

Dexter finds two thirds of the number.
Jack multiplies the number by $\frac{2}{3}$
Their new two-digit number has a digit total that is one more than that of their original number.

What number did they start with?
Show each step of their calculation.

They started with
24

Dexter:
$24 \div 3=8$
$8 \times 2=16$

Jack:
$24 \times 2=48$
$48 \div 3=16$

