White Rose Maths

Spring - Block 6

Ratio

Overview

Small Steps

- Using ratio language
- Ratio and fractions
- Introducing the ratio symbol
- Calculating ratio
- Using scale factors
- Calculating scale factors
- Ratio and proportion problems

NC Objectives

Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.

Solve problems involving similar shapes where the scale factor is known or can be found.

Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.

Using Ratio Language

Notes and Guidance

Children will understand that a ratio shows the relationship between two values and can describe how one is related to another.

They will start by making simple comparisons between two different quantities. For example, they may compare the number of boys to girls in the class and write statements such as, “For every one girl, there are two boys”.

Mathematical Talk

How would your sentences change if there were 2 more blue flowers?

How would your sentences change if there were 10 more pink flowers?

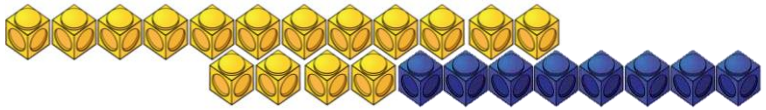
Can you write a “For every…” sentence for the number of boys and girls in your class?

Varied Fluency

Complete the sentences.

For every two blue flowers there are ____ pink flowers.
For every blue flower there are _____ pink flowers.

Use cubes to help you complete the sentences.

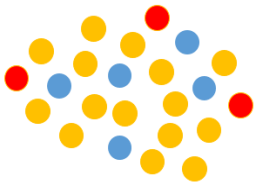


For every ____ , there are ____

For every 8 , there are ____

For every 1 , there are ____

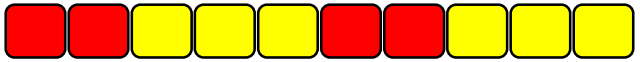
How many “For every…” sentences can you write to describe these counters?



Using Ratio Language

Reasoning and Problem Solving

Whitney lays tiles in the following pattern



If she has 16 red tiles and 20 yellow tiles remaining, can she continue her pattern without there being any tiles left over?

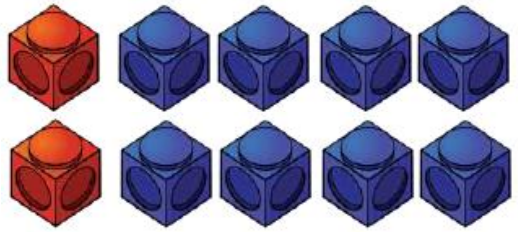
Explain why.

Possible responses:

For every two red tiles there are three yellow tiles. If Whitney continues the pattern she will need 16 red tiles and 24 yellow tiles. She cannot continue the pattern without there being tiles left over.

20 is not a multiple of 3

True or False?



- For every red cube there are 8 blue cubes.
- For every 4 blue cubes there is 1 red cube.
- For every 3 red cubes there would be 12 blue cubes.
- For every 16 cubes, 4 would be red and 12 would be blue.
- For every 20 cubes, 4 would be red and 16 would be blue.

False

True

True

False

True

Ratio and Fractions

Notes and Guidance

Children often think a ratio 1 : 2 is the same as a fraction of $\frac{1}{2}$

In this step, they use objects and diagrams to compare ratios and fractions.

Mathematical Talk

How many counters are there altogether?

How does this help you work out the fraction?

What does the denominator of the fraction tell you?

How can a bar model help you to show the mints and chocolates?

Varied Fluency

The ratio of red counters to blue counters is 1 : 2

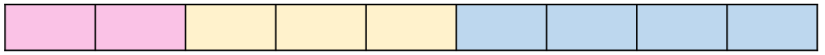
What fraction of the counters is blue?

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$

What fraction of the counters is red?

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$

This bar model shows the ratio 2 : 3 : 4



What fraction of the bar is pink?
What fraction of the bar is yellow?
What fraction of the bar is blue?

One third of the sweets in a box are mints.
The rest are chocolates.
What is the ratio of mints to chocolates in the box?

Ratio and Fractions

Reasoning and Problem Solving

Ron plants flowers in a flower bed.
For every 2 red roses he plants 5 white roses.

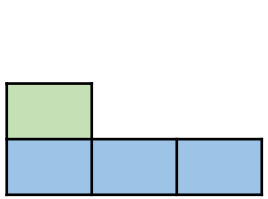
He says,

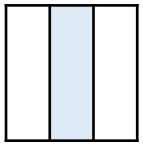
$\frac{2}{5}$ of the roses are red.

Is Ron correct?

Ron is incorrect because $\frac{2}{7}$ of the roses are red. He has mistaken a part with the whole.

Which is the odd one out?
Explain your answer.





 is the odd one out because one part out of three is a different colour. The others are one part out of four.

There are some red and green cubes in a bag. $\frac{2}{5}$ of the cubes are red.

True or False?

- For every 2 red cubes there are 5 green cubes.
- For every 2 red cubes there are 3 green cubes.
- For every 3 green cubes there are 2 red cubes.
- For every 3 green cubes there are 5 red cubes.

Explain your answers.

False

True

True

False

Introducing the Ratio Symbol

Notes and Guidance

Children are introduced to the colon notation as the ratio symbol, and continue to link this with the language 'for every..., there are...'
They need to read ratios e.g. 3 : 5 as "three to five".
Children understand that the notation relates to the order of parts. For example, 'For every 3 bananas there are 2 apples would be the same as 3 : 2 and for every 2 apples there are 3 bananas would be the same as 2 : 3

Mathematical Talk

What does the : symbol mean in the context of ratio?

Why is the order of the numbers important when we write ratios?

How do we write a ratio that compares three quantities?

How do we say the ratio "3 : 7"?

Varied Fluency

Complete:

The ratio of red counters to blue counters is ☐ : ☐

The ratio of blue counters to red counters is ☐ : ☐

Write down the ratio of:

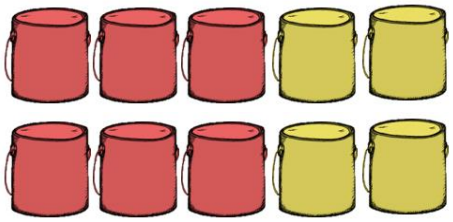
- Bananas to strawberries
- Blackberries to strawberries
- Strawberries to bananas to blackberries
- Blackberries to strawberries to bananas

The ratio of red to green marbles is 3 : 7
Draw an image to represent the marbles.
What fraction of the marbles are red?
What fraction of the marbles are green?

Introducing the Ratio Symbol

Reasoning and Problem Solving

Tick the correct statements.



- There are two yellow tins for every three red tins.
- There are two red tins for every three yellow tins.
- The ratio of red tins to yellow tins is 2 : 3
- The ratio of yellow tins to red tins is 2 : 3

Explain which statements are incorrect and why.

The first and last statement are correct. The other statements have the ratios the wrong way round.

In a box there are some red, blue and green pens.

The ratio of red pens to green pens is 3 : 5

For every 1 red pen there are two blue pens.

Write down the ratio of red pens to blue pens to green pens.

R : G

3 : 5

R : B

1 : 2 or

3 : 6

R : B : G

3 : 6 : 5

Calculating Ratio

Notes and Guidance

Children build on their knowledge of ratios and begin to calculate ratios. They answer worded questions in the form of 'for every... there are ...' and need to be able to find both a part and a whole.
They should be encouraged to draw bar models to represent their problems, and clearly label the information they have been given and what they want to calculate.

Mathematical Talk

How can we represent this ratio using a bar model?

What does each part represent? What will each part be worth?

How many parts are there altogether? What is each part worth?

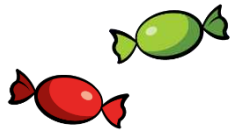
If we know what one part is worth, can we calculate the other parts?

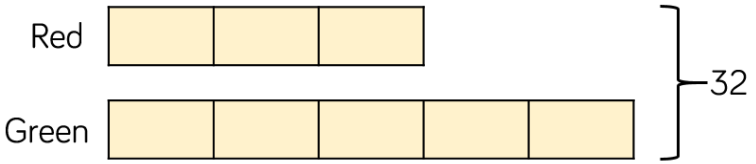
Varied Fluency

- A farmer plants some crops in a field.
For every 4 carrots he plants 2 leeks.
He plants 48 carrots in total.
How many leeks did he plant?
How many vegetables did he plant in total?

- Jack mixes 2 parts of red paint with 3 parts blue paint to make purple paint.
If he uses 12 parts blue paint, how many parts red paint does he use?

- Eva has a packet of sweets.
For every 3 red sweets there are 5 green sweets.
If there are 32 sweets in the packet in total, how many of each colour are there?
You can use a bar model to help you.





Calculating Ratio

Reasoning and Problem Solving

Teddy has two packets of sweets.

In the first packet, for every one strawberry sweet there are two orange sweets.

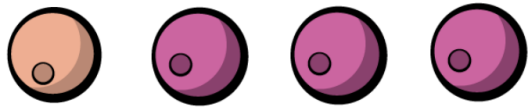
In the second packet, for every three orange sweets there are two strawberry sweets.

Each packet contains 15 sweets in total.

Which packet has more strawberry sweets and by how many?

The first packet has 5 strawberry sweets and 10 orange sweets.
The second packet has 6 strawberry sweets and 9 orange sweets.
The second packet has 1 more strawberry sweet than the first packet.

Annie is making some necklaces to sell.
For every one pink bead, she uses three purple beads.



Each necklace has 32 beads in total.

The cost of the string is £2.80
The cost of a pink bead is 72p.
The cost of a purple bead is 65p.

How much does it cost to make one necklace?

Each necklace has 8 pink beads and 24 purple beads.

The cost of the pink beads is £5.76

The cost of the purple beads is £15.60

The cost of a necklace is £24.16

Using Scale Factors

Notes and Guidance

In this step, children enlarge shapes to make them 2 or 3 times as big etc. They need to be introduced to the term "scale factor" as the name for this process.

Children should be able to draw 2-D shapes on a grid to a given scale factor and be able to use vocabulary, such as, "Shape A is three times as big as shape B".

Mathematical Talk

What does enlargement mean?

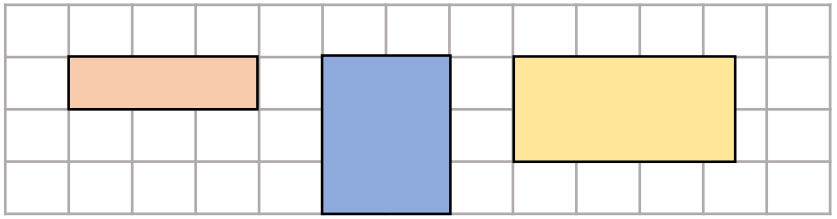
What does scale factor mean?

Why do we have to double/triple all the sides of each shape?

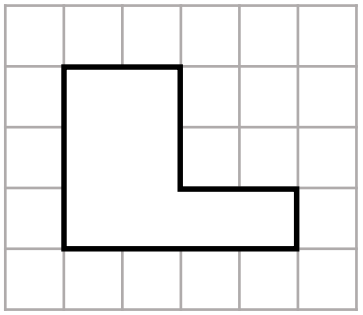
Have the angles changed size?

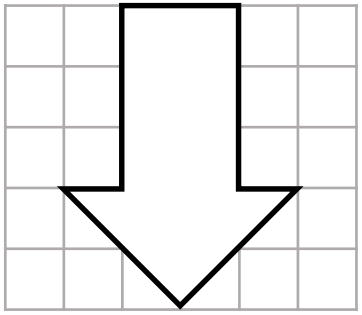
Varied Fluency

Copy these rectangles onto squared paper then draw them double the size, triple the size and 5 times as big.



Copy these shapes onto squared paper then draw them twice as big and three times as big.





Enlarge these shapes by:

- Scale factor 2
- Scale factor 3
- Scale factor 4

Using Scale Factors

Reasoning and Problem Solving

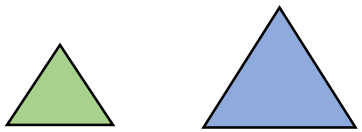
Draw a rectangle 3 cm by 4 cm.

Enlarge your rectangle by scale factor 2.

Compare the perimeter, area and angles of your two rectangles.

The perimeter has doubled, the area is four times as large, the angles have stayed the same.

Here are two equilateral triangles.
The blue triangle is three times larger than the green triangle.



(Not drawn to scale)

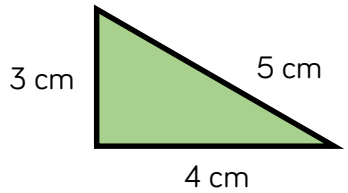
Find the perimeter of both triangles.

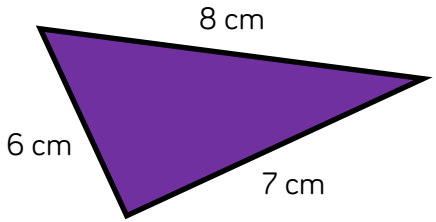
The blue triangle has a perimeter of 15 cm.

The green triangle has a perimeter of 5 cm.

Jack says:

The purple triangle is green triangle enlarged by scale factor 3





Do you agree?
Explain why.

Possible answer
I do not agree because Jack has increased the green shape by adding 3 cm to each side, not increasing it by a scale factor of 3

Calculating Scale Factors

Notes and Guidance

Children find scale factors when given similar shapes. They need to be taught that 'similar' in mathematics means that one shape is an exact enlargement of the other, not just they have some common properties.

Children use multiplication and division facts to calculate missing information and scale factors.

Mathematical Talk

What does similar mean?

What do you notice about the length/width of each shape?

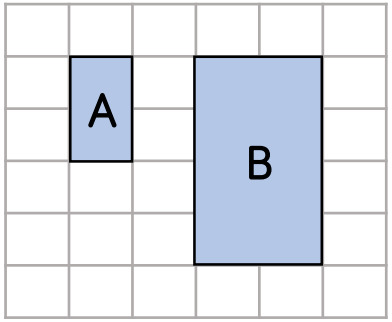
How would drawing the rectangles help you?

How much larger/smaller is shape A compared to shape B?

What does a scale factor of 2 mean? Can you have a scale factor of 2.5?

Varied Fluency

Complete the sentences.



Shape B is _____ as big as shape A.

Shape A has been enlarged by scale factor _____ to make shape B.

The rectangles described in the table are all similar to each other. Fill in the missing lengths and widths and complete the sentences.

Rectangle	Length	Width
A	5 cm	2 cm
B		4 cm
C	25 cm	
D		18 cm

From A to B, the scale factor of enlargement is ____
From A to C, the scale factor of enlargement is ____
From A to D the scale factor of enlargement is ____
From B to D, the scale factor of enlargement is ____

Calculating Scale Factors

Reasoning and Problem Solving

A rectangle has a perimeter of 16 cm.
An enlargement of this rectangle has a perimeter of 24 cm.

The length of the smaller rectangle is 6 cm.

Draw both rectangles.

Smaller rectangle:
length – 6 cm
width – 2 cm

Larger rectangle:
length – 9 cm
width – 3 cm

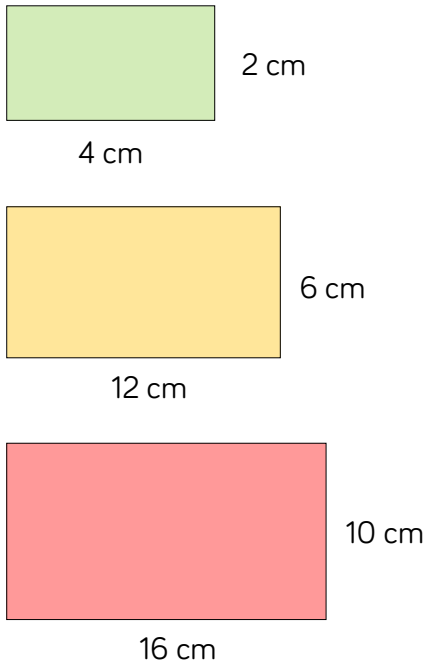
Scale factor: 1.5

Always, sometimes, or never true?

To enlarge a shape you just need to do the same thing to each of the sides.

Sometimes.
This only works when we are multiplying or dividing the lengths of the sides. It does not work when adding or subtracting.

Ron says that these three rectangles are similar.



Do you agree?
Explain your answer.

Ron is incorrect.
The orange rectangle is an enlargement of the green rectangle with scale factor 3.
The red rectangle, however, is not similar to the other two as the side lengths are not in the same ratio.

Ratio and Proportion Problems

Notes and Guidance

Children will apply the skills they have learnt in the previous steps to a wide range of problems in different contexts.

They may need support to see that different situations are in fact alternative uses of ratio.

Bar models will again provide valuable pictorial support.

Mathematical Talk

How does this problem relate to ratio?

Can we represent this ratio using a bar model?

What does each part represent? What is the whole?

What is the same about the ratios?

What is different about them?

Varied Fluency

How much of each ingredient is needed to make soup for:

- 3 people
- 9 people
- 1 person

What else could you work out?

Recipe for 6 people

- 1 onion
- 60 g butter
- 180 g lentils
- 1.2 litres stock
- 480 ml tomato juice

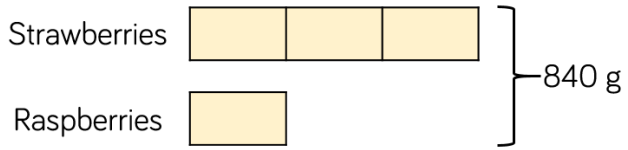
Two shops sell the same pens for these prices.

Safeway
4 pens £2.88

K-mart
7 pens £4.83

Which shop is better value for money?

The mass of strawberries in a smoothie is three times the mass of raspberries in the smoothie. The total mass of the fruit is 840 g. How much of each fruit is needed.



Ratio and Proportion Problems

Reasoning and Problem Solving

This recipe makes 10 flapjacks.

Flapjacks

120 g butter
100 g brown sugar
4 tablespoons golden syrup
250 g oats
40 g sultanas

Amir has 180 g butter.

What is the largest number of flapjacks he can make?

How much of the other ingredients will he need?

He has enough butter to make 15 flapjacks.
He will need 150 g brown soft sugar, 6 tablespoons golden syrup, 375 g oats and 60 g sultanas.

Alex has two packets of sweets.

In the first packet, for every 2 strawberry sweets there are 3 orange.

In the second packet, for one strawberry sweet, there are three orange.

Each packet has the same number of sweets.

The second packet contains 15 orange sweets.

How many strawberry sweets are in the first packet?

Second packet:
15 orange
5 strawberry.

So there are 20 sweets in each packet.

First packet:
8 strawberry
12 orange

The first packet contains 8 strawberry sweets.