White Rose Maths

Spring - Block 3

Fractions

Overview

Small Steps

- What is a fraction?
- Equivalent fractions (1)
- Equivalent fractions (2)
- Fractions greater than 1
- Count in fractions
- Add 2 or more fractions
- Subtract 2 fractions
- Subtract from whole amounts
- Calculate fractions of a quantity
- Problem solving – calculate quantities

NC Objectives

Recognise and show, using diagrams, families of common equivalent fractions.

Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.

Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number.

Add and subtract fractions with the same denominator.

What is a Fraction?

Notes and Guidance

Children explore fractions in different representations, for example, fractions of shapes, quantities and fractions on a number line.

They explore and recap the meaning of numerator and denominator, non-unit and unit fractions.

Mathematical Talk

How can we sort the fraction cards?

What fraction does each one represent?

Could some cards represent more than one fraction?

Is $\frac{1.5}{3}$ an example of a non-unit fraction? Why?

Using Cuisenaire, how many white rods are equal to an orange rod? How does this help us work out what fraction the white rod represents?

Varied Fluency

Here are 9 cards.
Sort the cards into different groups.
Can you explain how you made your decision?
Can you sort the cards in a different way?
Can you explain how your partner has sorted the cards?
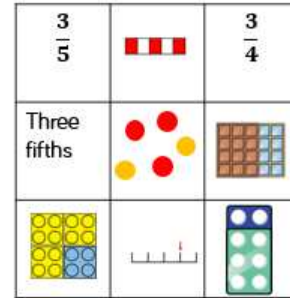
Complete the Frayer model to describe a unit fraction.

Can you use the model to describe the following terms?

Non-unit fraction   Numerator   Denominator

Definition	Characteristics
	unit fraction
Examples	Non-examples
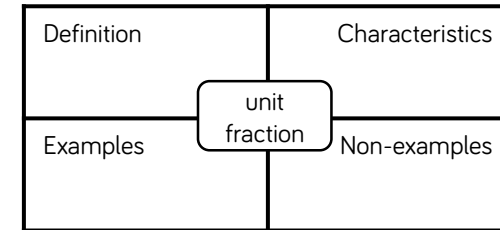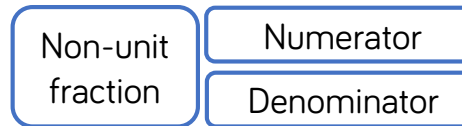
Use Cuisenaire rods.
If the orange rod is one whole, what fraction is represented by:
- The white rod   -   The red rod
- The yellow rod   -   The brown rod

Choose a different rod to represent one whole.; what do the other rods represent now?

What is a Fraction?

Reasoning and Problem Solving

Always, Sometimes, Never?

Alex says,

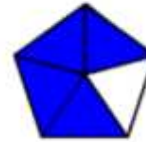*If I split a shape into 4 parts, I have split it into quarters.*
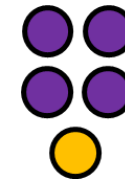
Explain your answer.

Sometimes

If the shape is not split equally, it will not be in quarters.

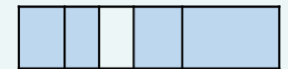Which representations of $\frac{4}{5}$ are incorrect?

$$\frac{4}{5}$$

Explain how you know.

The image of the dogs could represent $\frac{2}{5}$ or $\frac{3}{5}$

The bar model is not divided into equal parts so this does not represent $\frac{4}{5}$

Equivalent Fractions (1)

Notes and Guidance

Children use strip diagrams to investigate and record equivalent fractions.

They start by comparing two fractions before moving on to finding more than one equivalent fraction on a fraction wall.

Mathematical Talk

Look at the equivalent fractions you have found.
What relationship can you see between the numerators and denominators? Are there any patterns?

Can a fraction have more than one equivalent fraction?

Can you use Cuisenaire rods or pattern blocks to investigate equivalent fractions?

Varied Fluency

Use two strips of equal sized paper.
Fold one strip into quarters and the other into eighths.
Place the quarters on top of the eighths and lift up one quarter; how many eighths can you see?
How many eighths are equivalent to one quarter?
Which other equivalent fractions can you find?

Using squared paper, investigate equivalent fractions using equal parts e.g. $\frac{2}{4} = \frac{?}{8}$
Start by drawing a bar 8 squares long.
Underneath, compare the same length bar split into four equal parts.

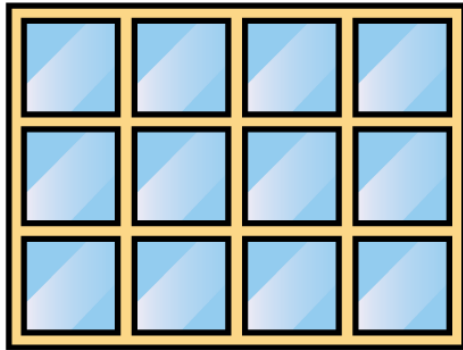How many fractions that are equivalent to one half can you see on the fraction wall?
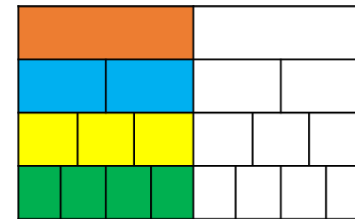
Draw extra rows to show other equivalent fractions.

How many equivalent fractions can you see in this picture?

Children can give a variety of possibilities. Examples:

$$\frac{1}{2} = \frac{6}{12} = \frac{3}{6}$$

$$\frac{1}{4} = \frac{3}{12}$$

Eva says,

I know that $\frac{3}{4}$ is equivalent to $\frac{3}{8}$ because the numerators are the same.

Is Eva correct?
Explain why.

Eva is not correct. $\frac{3}{4}$ is equivalent to $\frac{6}{8}$ When the numerators are the same, the larger the denominator, the smaller the fraction.

Ron has two strips of the same sized paper.
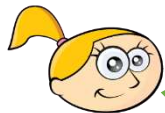He folds the strips into different sized fractions.
He shades in three equal parts on one strip and six equal parts on the other strip.
The shaded areas are equal.

What fractions could he have folded his strips into?

Ron could have folded his strips into sixths and twelfths, quarters and eighths or any other fractions where one of the denominators is double the other.

Equivalent Fractions (2)

Notes and Guidance

Children continue to understand equivalence through diagrams. They move onto using proportional reasoning to find equivalent fractions.

Attention should be drawn to the method of multiplying the numerators and denominators by the same number to ensure that fractions are equivalent.

Mathematical Talk

What other equivalent fractions can you find using the diagram?

What relationships can you see between the fractions?

If I multiply the numerator by a number, what do I have to do to the denominator to keep it equivalent? Is this always true?
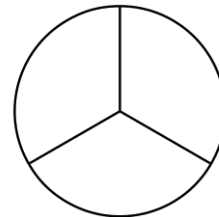
What relationships can you see between the numerator and denominator?

Varied Fluency

Using the diagram, complete the equivalent fractions.

$$\frac{1}{4} = \frac{\square}{12} \qquad \frac{1}{\square} = \frac{6}{12} \qquad \frac{2}{3} = \frac{\square}{12} \qquad \frac{5}{12} = \frac{\square}{24}$$

Using the diagram, complete the equivalent fractions.

$$\frac{1}{3} = \frac{\square}{6} = \frac{\square}{12} = \frac{\square}{24}$$

Complete:

$$\frac{1}{4} = \frac{2}{\square} = \frac{\square}{12} = \frac{4}{\square} = \frac{\square}{100} = \frac{\square}{500}$$

Equivalent Fractions (2)

Reasoning and Problem Solving

Tommy is finding equivalent fractions.

$$\frac{3}{4} = \frac{5}{6} = \frac{7}{8} = \frac{9}{10}$$

He says,

I did the same thing to the numerator and the denominator so my fractions are equivalent.

Do you agree with Tommy? Explain your answer.

Tommy is wrong. He has added two to the numerator and denominator each time. When you find equivalent fractions you either need to multiply or divide the numerator and denominator by the same number.

Use the digit cards to complete the equivalent fractions.

1 2 3
4 6 8

$$\frac{\square}{\square} = \frac{\square}{\square}$$
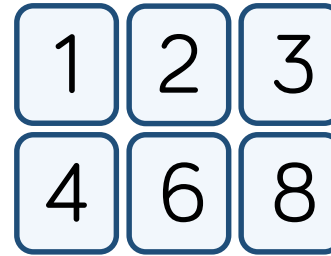
How many different ways can you find?

Possible answers:

$\frac{1}{2} = \frac{3}{6}$, $\frac{1}{2} = \frac{4}{8}$,

$\frac{1}{3} = \frac{2}{6}$, $\frac{1}{4} = \frac{2}{8}$,

$\frac{3}{4} = \frac{6}{8}$, $\frac{2}{3} = \frac{4}{6}$

Fractions Greater than 1

Notes and Guidance

Children use manipulatives and diagrams to show that a fraction can be split into wholes and parts.

Children focus on how many equal parts make a whole dependent on the number of equal parts altogether. This learning will lead on to Year 5 where children learn about improper fractions and mixed numbers.

Mathematical Talk

How many ____ make a whole?

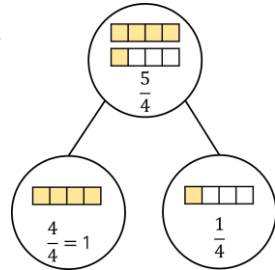If I have ____ eighths, how many more do I need to make a whole?

What do you notice about the numerator and denominator when a fraction is equivalent to a whole?
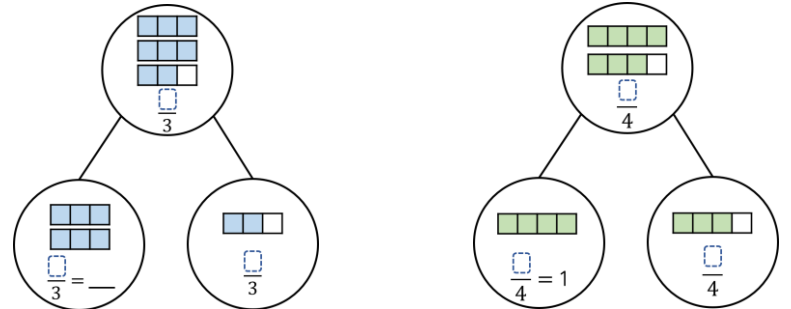
Varied Fluency

Complete the part-whole models and sentences.

There are ____ quarters altogether.

___ quarters = ____ whole and ____ quarter.

Write sentences to describe these part-whole models.

Complete. You may use part-whole models to help you.

$$\frac{10}{3} = \frac{9}{3} + \frac{\square}{3} = 3\frac{\square}{3}$$

$$\frac{\square}{3} = \frac{6}{3} + \frac{2}{3} = \square\frac{2}{3}$$

$$\frac{\square}{8} = \frac{16}{8} + \frac{3}{8} = \square\frac{\square}{\square}$$
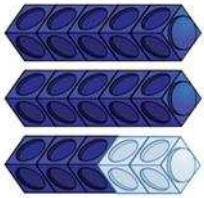
Reasoning and Problem Solving

3 friends share some pizzas.
Each pizza is cut into 8 equal slices.
Altogether, they eat 25 slices.
How many whole pizzas do they eat?

They eat 3 whole pizzas and 1 more slice.

Spot the mistake.

$\frac{13}{5}$ = 10 wholes and 3 fifths

There are 2 wholes not 10

$\frac{10}{5}$ = 2 wholes

$\frac{13}{5}$ = 2 wholes and 3 fifths

Rosie says,

$\frac{16}{4}$ is greater than $\frac{8}{2}$ because 16 is greater than 8

Do you agree?
Explain why.

I disagree with Rosie because both fractions are equivalent to 4

Children may choose to build both fractions using cubes, or draw bar models.

Count in Fractions

Notes and Guidance

Children explore fractions greater than one on a number line and start to make connections between improper and mixed numbers.

They use cubes and bar models to represent fractions greater than a whole. This will support children when adding and subtracting fractions greater than a whole.
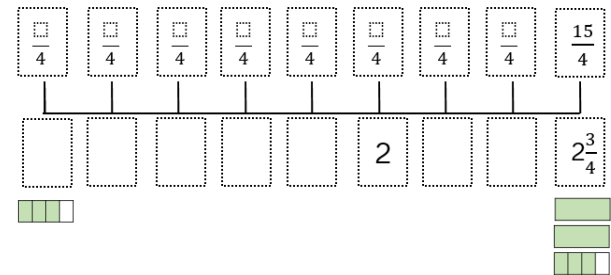
Mathematical Talk

How many ____ make a whole?

Can you write the missing fractions in more than one way?
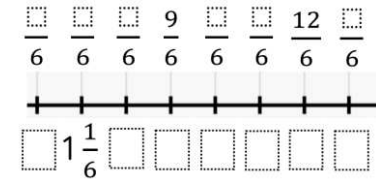
Are the fractions ascending or descending?

Varied Fluency

Complete the number line.

Draw bar models to represent each fraction.

Fill in the blanks using cubes or bar models to help you.
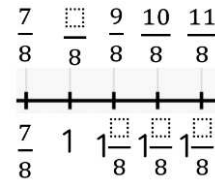
Write the next two fractions in each sequence.

a) $\dfrac{12}{7}, \dfrac{11}{7}, \dfrac{10}{7}, \underline{\quad}, \underline{\quad}$ b) $3\dfrac{1}{3}, 3, 2\dfrac{2}{3}, \underline{\quad}, \underline{\quad}$

c) $\dfrac{4}{11}, \dfrac{6}{11}, \dfrac{8}{11}, \underline{\quad}, \underline{\quad}$ d) $12\dfrac{3}{5}, 13\dfrac{1}{5}, 13\dfrac{4}{5}, \underline{\quad}, \underline{\quad}$

Count in Fractions

Reasoning and Problem Solving

Here is a number sequence.

$$\frac{5}{12}, \frac{7}{12}, \frac{10}{12}, \frac{14}{12}, \frac{19}{12}, \underline{}$$

Which fraction would come next?
Can you write the fraction in more than one way?

The fractions are increasing by one more twelfth each time. The next fraction would be $\frac{25}{12}$

Circle and correct the mistakes in the sequences.

$$\frac{5}{12}, \frac{8}{12}, \frac{11}{12}, \frac{15}{12}, \frac{17}{12}$$

$$\frac{9}{10}, \frac{7}{10}, \frac{6}{10}, \frac{3}{10}, \frac{1}{10}$$

$$\frac{5}{12}, \frac{8}{12}, \frac{11}{12}, \boxed{\frac{14}{12}}, \frac{17}{12}$$

$$\frac{9}{10}, \frac{7}{10}, \boxed{\frac{5}{10}}, \frac{3}{10}, \frac{1}{10}$$

Play the fraction game for four players.
Place the four fraction cards on the floor.
Each player stands in front of a fraction.
We are going to count up in tenths starting at 0
When you say a fraction, place your foot on your fraction.

$$\boxed{\frac{1}{10}} \qquad \boxed{\frac{2}{10}}$$

$$\boxed{\frac{3}{10}} \qquad \boxed{\frac{5}{10}}$$
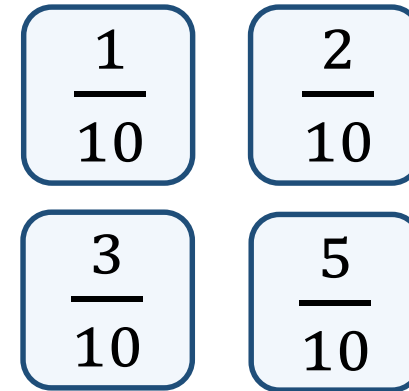
How can we make 4 tenths?
What is the highest fraction we can count to?
How about if we used two feet?

2 children can make four tenths by stepping on one tenth and three tenths at the same time. Alternatively, one child can make four tenths by stepping on $\frac{2}{10}$ with 2 feet. With one foot, they can count up to 11 tenths or one and one tenth. With two feet they can count up to 22 tenths.

Add 2 or More Fractions

Notes and Guidance

Children use practical equipment and pictorial representations to add two or more fractions. Children record their answers as an improper fraction when the total is more than 1

A common misconception is to add the denominators as well as the numerators. Use bar models to support children's understanding of why this is incorrect.

Children can also explore adding fractions more efficiently by using known facts or number bonds to help them.

Mathematical Talk

How many equal parts is the whole split into? How many equal parts am I adding?

Which bar model do you prefer when adding fractions? Why?

Can you combine any pairs of fractions to make one whole when you are adding three fractions?

Varied Fluency

■ Take two identical strips of paper.
Fold your paper into quarters.
Can you use the strips to solve

$\frac{1}{4} + \frac{1}{4}$ $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ $\frac{3}{4} + \frac{3}{4}$ $\frac{\square}{4} + \frac{\square}{4} = \frac{7}{4}$
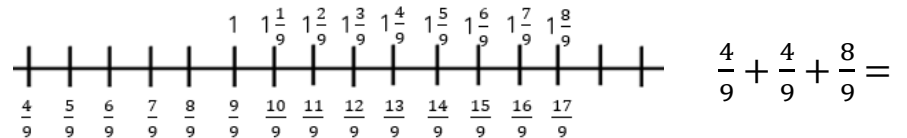
What other fractions can you make and add?

■ Use the models to add the fractions:

$\frac{2}{7} + \frac{2}{7} =$

$\frac{3}{5} + \frac{4}{5} =$

Choose your preferred model to add:

$\frac{2}{5} + \frac{1}{5}$ $\frac{3}{7} + \frac{6}{7}$ $\frac{7}{9} + \frac{4}{9}$

■ Use the number line to add the fractions.

$\frac{4}{9} + \frac{4}{9} + \frac{8}{9} =$

$\frac{4}{9} + \frac{5}{9} + \frac{8}{9}$ $\frac{1}{9} + \frac{11}{9} + 1$ $\frac{\square}{9} + \frac{5}{9} + \frac{7}{9} = \frac{17}{9}$

Add 2 or More Fractions

Reasoning and Problem Solving

Alex is adding fractions.

$$\frac{3}{9} + \frac{2}{9} = \frac{5}{18}$$

Is she correct? Explain why.

Alex is incorrect. Alex has added the denominators as well as the numerators.

How many different ways can you find to solve the calculation?

$$\frac{\Box}{\Box} + \frac{\Box}{\Box} = \frac{11}{9}$$

Any combination of ninths where the numerators total 11.

Mo and Teddy are solving:

$$\frac{6}{13} + \frac{5}{13} + \frac{7}{13}$$

Mo

The answer is 1 and $\frac{5}{13}$

Teddy

The answer is $\frac{18}{13}$

Who do you agree with? Explain why.

They are both correct. Mo has added $\frac{6}{13} + \frac{7}{13}$ to make 1 whole and then added $\frac{5}{13}$

Subtract 2 Fractions

Notes and Guidance

Children use practical equipment and pictorial representations to subtract fractions with the same denominator.

Encourage children to explore subtraction as take away and as difference. Difference can be represented on a bar model by using a comparison model and making both fractions in the subtraction.

Mathematical Talk

Have you used take away or difference to subtract the eighths using the strips of paper? How are they the same? How are they different?

How can I find a missing number in a subtraction? Can you count on to find the difference?
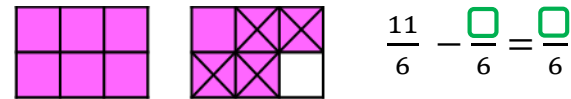
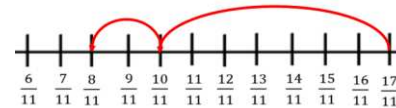Can I partition my fraction to help me subtract?

Varied Fluency

Use identical strips of paper and fold them into eighths. Use the strips to solve the calculations.

$$\frac{8}{8} - \frac{3}{8} = \qquad \frac{7}{8} - \frac{3}{8} = \qquad \frac{16}{8} - \frac{9}{8} = \qquad \frac{13}{8} - \frac{\square}{8} = \frac{7}{8}$$

Use the bar models to subtract the fractions.

$$\frac{6}{7} - \frac{2}{7} =$$

$$\frac{11}{6} - \frac{\square}{6} = \frac{\square}{6}$$

$$\frac{13}{5} - \frac{\square}{5} = \frac{6}{5}$$

Annie uses the number line to solve $\frac{17}{11} - \frac{9}{11}$

Use a number line to solve:

$$\frac{16}{13} - \frac{9}{13} \qquad \frac{16}{9} - \frac{9}{9} \qquad \frac{16}{7} - \frac{9}{7} \qquad \frac{16}{16} - \frac{9}{16}$$

16

Subtract 2 Fractions

Reasoning and Problem Solving

Match the number stories to the correct calculations.

Teddy eats $\frac{7}{8}$ of a pizza. Dora eats $\frac{4}{8}$ How much do they eat altogether?	$\frac{7}{8} + \frac{3}{8} = -$
Teddy eats $\frac{7}{8}$ of a pizza. Dora eats $\frac{4}{8}$ less. How much do they eat altogether?	$\frac{7}{8} + \frac{4}{8} = -$
Teddy eats $\frac{7}{8}$ of a pizza. Dora eats $\frac{3}{8}$ less. How much does Dora eat?	$\frac{7}{8} - \frac{3}{8} = -$

How many different ways can you find to solve the calculation?

$$\frac{\square}{7} - \frac{3}{7} = \frac{\square}{7} + \frac{\square}{7}$$

$$\frac{\square}{7} - \frac{3}{7} = \frac{\square}{7} - \frac{\square}{7}$$

Annie and Amir are working out the answer to this problem.

$$\frac{7}{9} - \frac{3}{9}$$

Annie uses this model.

Amir uses this model.

Which model is correct? Explain why.

Can you write a number story for each model?

Subtract from Whole Amounts

Notes and Guidance

Children continue to use practical equipment and pictorial representations to subtract fractions.

Children subtract fractions from a whole amount. Children need to understand how many equal parts are equivalent to a whole e.g. $\frac{9}{9} = 1$, $\frac{18}{9} = 2$ etc.

Mathematical Talk

What do you notice about the numerator and denominator when a fraction is equal to one whole?
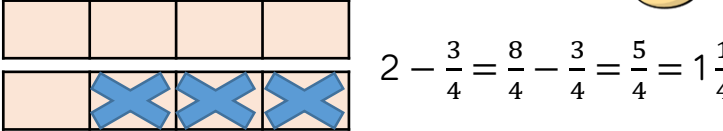
Using Jack's method, what's the same about your bar models? What's different?

How many more thirds/quarters/ninths do you need to make one whole?

Varied Fluency

Use cubes, strips of paper or a bar model to solve:

$$\frac{9}{9} - \frac{4}{9} = \frac{\square}{9} \qquad \frac{9}{9} - \frac{\square}{9} = \frac{2}{9} \qquad \frac{13}{9} - \frac{9}{9} = \frac{\square}{9}$$
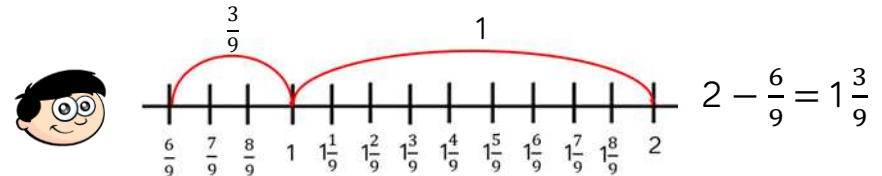
What's the same? What's different?

Jack uses a bar model to subtract fractions.

$$2 - \frac{3}{4} = \frac{8}{4} - \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}$$

Use Jack's method to calculate.

$$3 - \frac{3}{4} = \qquad 3 - \frac{3}{8} = \qquad 3 - \frac{7}{8} = \qquad 3 - \frac{15}{8} =$$

Dexter uses a number line to find the difference between 2 and $\frac{6}{9}$

$$2 - \frac{6}{9} = 1\frac{3}{9}$$

Use a number line to find the difference between:

2 and $\frac{2}{3}$ \qquad 2 and $\frac{2}{5}$ \qquad $\frac{2}{5}$ and 4

Subtract from Whole Amounts

Reasoning and Problem Solving

Dora is subtracting a fraction from a whole.

$$5 - \frac{3}{7} = \frac{2}{7}$$

Can you spot her mistake?

What should the answer be?

How many ways can you make the statement correct?

$$2 - \frac{\square}{8} = \frac{5}{8} + \frac{\square}{8}$$

Dora has not recognised that 5 is equivalent to $\frac{35}{7}$

$$5 - \frac{3}{7} = \frac{33}{7} = 4\frac{5}{7}$$

Lots of possible responses.

e.g.

$$2 - \frac{1}{8} = \frac{5}{8} + \frac{10}{8}$$

$$2 - \frac{7}{8} = \frac{5}{8} + \frac{4}{8}$$

$$2 - \frac{9}{8} = \frac{5}{8} + \frac{2}{8}$$

Whitney has a piece of ribbon that is 3 metres long.

She cuts it into 12 equal pieces and gives Teddy 3 pieces.

How many metres of ribbon does Whitney have left?

Cutting 3 metres of ribbon into 12 pieces means each metre of ribbon will be in 4 equal pieces. Whitney will have $\frac{12}{4}$ to begin with.

$$\frac{12}{4} - \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$$

Whitney has $2\frac{1}{4}$ metres of ribbon left.

Fractions of a Quantity

Notes and Guidance

Children use their knowledge of finding unit fractions of a quantity, to find non-unit fractions of a quantity.

They use concrete and pictorial representations to support their understanding. Children link bar modelling to the abstract method in order to understand why the method works.

Mathematical Talk

What is the whole? What fraction of the whole are we finding? How many equal parts will I divide the whole into?

What's the same and what's different about the calculations? Can you notice a pattern?

What fraction of her chocolate bar does Whitney have left? How many grams does she have left? Can you represent this on a bar model?
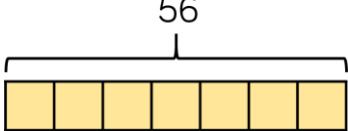
Varied Fluency

Mo has 12 apples.
Use counters to represent his apples and find:

$\frac{1}{2}$ of 12 $\frac{1}{4}$ of 12 $\frac{1}{3}$ of 12 $\frac{1}{6}$ of 12

Now calculate:

$\frac{2}{2}$ of 12 $\frac{3}{4}$ of 12 $\frac{2}{3}$ of 12 $\frac{5}{6}$ of 12

What do you notice? What's the same and what's different?

Use a bar model to help you represent and find:

$\frac{1}{7}$ of $56 = 56 \div \square$

56

$\frac{2}{7}$ of 56 $\frac{3}{7}$ of 56 $\frac{4}{7}$ of 56 $\frac{4}{7}$ of 28 $\frac{7}{7}$ of 28

Whitney eats $\frac{3}{8}$ of 240 g bar of chocolate.
How many grams of chocolate has she eaten?

Fractions of a Quantity

Reasoning and Problem Solving

True or False?

To find $\frac{3}{8}$ of a number, divide by 3 and multiply by 8

Convince me.

False.
Divide the whole by 8 to find one eighth and then multiply by three to find three eighths of a number.

Ron gives $\frac{2}{9}$ of a bag of 54 marbles to Alex.

Teddy gives $\frac{3}{4}$ of a bag of marbles to Alex.

Ron gives Alex more marbles than Teddy.

How many marbles could Teddy have to begin with?

$$\frac{2}{9} \text{ of } 54 > \frac{3}{4} \text{ of } \boxed{}$$

Teddy could have 16, 12, 8 or 4 marbles to begin with.

Calculate Quantities

Notes and Guidance

Children solve more complex problems for fractions of a quantity. They continue to use practical equipment and pictorial representations to help them see the relationships between the fraction and the whole.

Encourage children to use the bar model to solve word problems and represent the formal method.
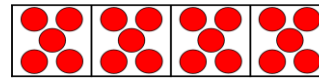
Mathematical Talk

If I know one quarter of a number, how can I find three quarters of a number?

If I know one of the equal parts, how can I find the whole?
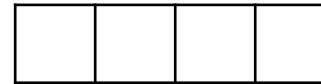
How can a bar model support my working?

Varied Fluency

◻ Use the counters and bar models to calculate the whole:

There are ____ counters in one part.

$\frac{1}{4} =$ ____ $\frac{2}{4} =$ ____ $\frac{3}{4} =$ ____ $\frac{4}{4}$ or 1 whole = ____

There are 7 counters in one part.

$\frac{1}{4} =$ ____ $\frac{2}{4} =$ ____ $\frac{3}{4} =$ ____ $\frac{4}{4}$ or 1 whole = ____

◻ Complete.

Whole	Unit Fraction	Non-unit Fraction
The whole is 24	$\frac{1}{6}$ of 24 = ____	$\frac{5}{6}$ of 24 = ____
The whole is ____	$\frac{1}{3}$ of ____ = 30	$\frac{2}{3}$ of ____ = ____
The whole is ____	$\frac{1}{5}$ of ____ = 30	$\frac{3}{5}$ of ____ = ____

◻ Jack has a bottle of lemonade.
He has one-fifth left in the bottle.
There are 150 ml left.
How much lemonade was in the bottle when it was full?

Calculate Quantities

Reasoning and Problem Solving

The school kitchen needs to buy carrots for lunch.
A large bag has 200 carrots and a medium bag has $\frac{3}{5}$ of a large bag.
Mrs Rose says,

I need 150 carrots so I will have to buy a large bag.
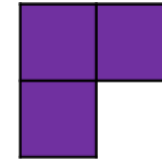
Is Mrs Rose correct?
Explain your reasoning.

Mrs Rose is correct.
$\frac{3}{5}$ of 200 = 120
Mrs Rose will need a large bag.

These three squares are $\frac{1}{4}$ of a whole shape.

How many different shapes can you draw that could be the complete shape?

Lots of different possibilities. The shape should have 12 squares in total.

If $\frac{1}{8}$ of A = 12, find the value of A, B and C.

$$\frac{5}{8} \text{ of A} = \frac{3}{4} \text{ of B} = \frac{1}{6} \text{ of C}$$

A = 96
B = 80
C = 360